New Aspects of Heterotic Geometry

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He Feng, arXiv:1607.04628

J. Gray, P. Oehlmann, and N. Raghuram, in progress

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Heterotic Geometry

- Heterotic EFT is closely tied to difficult questions in the geometry of Calabi-Yau manifolds, X, and vector bundles, $\pi : V \to X$:
  - SUSY $\to$ bundle stability and holomorphy
  - Form of the 4D potential $\to$ D-terms, Holomorphic Chern Simons theory, GKV Superpotential: $W \sim \int_X H \wedge \Omega$ with $H \sim dB - \omega_{3YM} + \omega_{3L}$
  - Massless spectra/couplings $\to$ bundle valued cohomology, Yoneda products
- Difficult to "engineer" 4D theory of choice $\Rightarrow$ large scale scans of pairs (X, V)
- Better "control" of geometry of heterotic bundles/manifolds would be very helpful
- Here control = better dictionary linking EFT and geometry.

Work in progress...

- This talk: a repackaging of heterotic geometry that may shed light on redundancies in the space of heterotic vacua and make it easier to find models with given spectra, moduli,etc.
- Two hints of structure inspired this work:
- The heterotic moduli space naturally combines fluctuations of the background manifold and gauge fields (e.g. LA, Gray, Ovrut, Lukas, Sharpe, de la Ossa, Svanes, Hardy, Candelas, McOrist...).
- Heterotic "dualities" naturally mix moduli (and d.o.f) associated to manifolds and bundles (e.g. Distler, Kachru, Blumenhagen, Rahn, LA, Feng, etc)

Heterotic redundancies

- Want to better understand intriguing $(0,2)$ GLSM "Duality" from the '90s...
- Target Space Duality: Two $(0,2)$ GLSMs which share a non-geometric (i.e. LG) vacuum. In this case, the two large volume limits (i.e. (X, V) and $(\tilde{X}, \tilde{V})$ give same apparent effective 4D spectrum (Distler, Kachru, Blumenhagen...):

$$h^*(X, \wedge^k V) = h^*(\widetilde{X}, \wedge^k \widetilde{V}), \qquad k = 1, 2, \cdots, rk(V)$$

$$h^{2,1}(X) + h^{1,1}(X) + h^1_X(End_0(V)) = h^{2,1}(\widetilde{X}) + h^{1,1}(\widetilde{X}) + h^1_X(End_0(\widetilde{V}))$$

- Different manifolds and vector bundles, but same physics?
- Landscape study: Blumenhagen and Rahn created $\sim 83,000$ TSD pairs and nearly all produced same 4D spectra ($\sim 90\%$)

Questions

- Currently GLSM combinatorics lead to theories with the same spectra. Are they actually the same NLSM? Possibilities:
  - $(0, 2)$ "Mirrors"? $\Leftrightarrow$ Same sigma models, different geometries.
  - $(0, 2)$ Geometric transitions? (i.e. heterotic conifolds/flops). Branch structure in vacuum space?
  - Practically powerful tool (Might make it easier to find/characterize "interesting" heterotic vacua...). Remove redundancy.
  - Can this be understood purely in terms of geometry? $(X, V) \leftrightarrow (\tilde{X}, \tilde{V})$??
  - GLSM/NLSM and geometric understandings desired. Here will focus on the latter...

$(0,2)$ GLSMs in a nutshell

- Abelian, massive 2D theory $\stackrel{\text{IR flow}}{\longrightarrow}$ $(0,2)$ CFT
- $U(1)$ gauge fields $A^{(\alpha)}$, $\alpha = 1, \ldots r$
- Chiral superfields: $\{X_i | i = 1, \ldots d\}$ charge $(Q_i^{(\alpha)})$, $\{P_l | l = 1, \ldots \gamma\}$, charge $(-M_l^{\alpha})$.
- Fermi superfields: $\{\Lambda^a | a = 1 \ldots, \delta\}$ charge $N_a^{(\alpha)}$, $\{\Gamma_j^{(\alpha)} | j = 1 \ldots c\}$ charge $(-S_j^{(\alpha)})$.
- Gauge and gravitational anomaly cancellation:

$$\sum_{a=1}^{\delta} N_a^{(\alpha)} = \sum_{l=1}^{\gamma} M_l^{(\alpha)} \qquad \sum_{i=1}^{d} Q_i^{(\alpha)} = \sum_{j=1}^{c} S_j^{(\alpha)}$$

$$\sum_{l=1}^{\gamma} M_l^{(\alpha)} M_l^{(\beta)} - \sum_{a=1}^{\delta} N_a^{(\alpha)} N_a^{(\beta)} = \sum_{j=1}^{c} S_j^{(\alpha)} S_j^{(\beta)} - \sum_{i=1}^{d} Q_i^{(\alpha)} Q_i^{(\beta)}$$

for all $\alpha, \beta = 1, ..., r$.

We encapsulate all this information in a table:

X_i				Γ^j			
$Q_1^{(1)}$	$Q_2^{(1)}$	$\dots$	$Q_d^{(1)}$	$-S_1^{(1)}$	$-S_2^{(1)}$	$\dots$	$S_c^{(1)}$
$Q_1^{(2)}$	$Q_2^{(2)}$	$\dots$	$Q_d^{(2)}$	$-S_1^{(2)}$	$-S_2^{(2)}$	$\dots$	$S_c^{(2)}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$Q_1^{(r)}$	$Q_2^{(r)}$	$\dots$	$Q_d^{(r)}$	$-S_1^{(r)}$	$-S_2^{(r)}$	$\dots$	$S_c^{(r)}$

Λ^a				P_l			
$N_1^{(1)}$	$N_2^{(1)}$	$\dots$	$N_\delta^{(1)}$	$-M_1^{(1)}$	$-M_2^{(1)}$	$\dots$	$-M_\gamma^{(1)}$
$N_1^{(2)}$	$N_2^{(2)}$	$\dots$	$N_\delta^{(2)}$	$-M_1^{(2)}$	$-M_2^{(2)}$	$\dots$	$-M_\gamma^{(2)}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$N_1^{(r)}$	$N_2^{(r)}$	$\dots$	$N_\delta^{(r)}$	$-M_1^{(r)}$	$-M_2^{(r)}$	$\dots$	$-M_\gamma^{(r)}$

The GLSM potential

- Superpotential: $S = \int d^2 z d\theta \left[\sum_j \Gamma^j G_j(X_i) + \sum_{l,a} P_l \Lambda^a F_a^l(X_i) \right]$
- G_j and F_a^l are quasi-homogeneous polynomials w/ multi-degrees:

G^j			
S_1	S_2	$\dots$	S_c

$F_a{}^l$			
$M_1 - N_1$	$M_1 - N_2$	$\dots$	$M_1 - N_\delta$
$M_2 - N_1$	$M_2 - N_2$	$\dots$	$M_2 - N_\delta$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$M_\gamma - N_1$	$M_\gamma - N_2$	$\dots$	$M_\gamma - N_\delta$

- F-term: $V_F = \sum_j \left| G_j(x_i) \right|^2 + \sum_a \left| \sum_l p_l F_a^l(x_i) \right|^2$
- D-term: $V_D = \sum_{\alpha=1}^{r} \left(\sum_{i=1}^{d} Q_i^{(\alpha)} |x_i|^2 - \sum_{l=1}^{\gamma} M_l^{(\alpha)} |p_l|^2 - \xi^{(\alpha)} \right)^2$
- Transversality condition: $F_a^l(x_i) = 0$ only when $x_i = 0 \; \forall i$
- FI Parameter ($\xi^{(\alpha)} \in \mathbb{R}$) controls the *phases*

- E.g. $\xi > 0 \Rightarrow G_j(X_i) = 0$ and $\langle P \rangle = 0 \Rightarrow$ Geometric phase.
- Geometry: (X, V) with X a CY and bundle described via a monad:

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r_{\mathcal{V}}} \xrightarrow{\otimes E_i^a} \oplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a^l} \oplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \to 0$$

with $V = \frac{ker(F_a^l)}{im(E_i^a)}$

- E.g. $\xi < 0 \Rightarrow \langle p \rangle \neq 0 \Rightarrow$ Non-geometric phase
- E.g. Landau-Ginzburg orbifold w. superpotential:

$$\mathcal{W}(X_i, \Lambda^a, \Gamma^i) = \sum_j \Gamma^j G_j(X_i) + \sum_a \Lambda^a F_a(X_i)$$

- With multiple $U(1)$s, hybrid phases.

Target space duality (TSD)

- Observation (Distler, Kachru): In LG-phase, G and F on equal footing. Could be interchanged...$\sum_j \Gamma^j G_j(X_i) + \sum_a \Lambda^a F_a(X_i)$
- Algorithm: Find phase with one $\langle p_l \rangle \neq 0$ for some l.
- Rescale: $\tilde{\Lambda}^{a_i} := \frac{\Gamma^{j_i}}{\langle p_1 \rangle}$, $\tilde{\Gamma}^{j_i} := \langle p_1 \rangle \Lambda^{a_i}$ $\forall i = 1, \ldots k$ s.t. $\sum_i ||G_{j_i}|| = \sum_i ||{F_{a_i}}^1||$
- Move to a region in bundle moduli space where Λ^{a_i} appear only with P_1 $\forall i \Rightarrow F^l_{a_i} = 0$ $\forall l \neq 1$, $i = 1, \ldots k$.
- Leave non-geometric phase and define new Fermi superfields s.t. $||\tilde{\Lambda}^{a_i}|| = ||\Gamma^{j_i}|| - ||P_1||$ and $||\tilde{\Gamma}^{j_i}|| = ||\Lambda^{a_i}|| + ||P_1||$.
- Return to a generic pt. in moduli space to define new TS dual $(0,2)$ GLSM w/ new geometric phase: $(\widetilde{X}, \widetilde{V})$.

Example

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−2	−2	1	0	0	2	−3
1	1	1	2	2	2	0	−4	−5	0	1	1	6	−8

- $SU(3)$ bundle with
  $dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 2 + 68 + 322 = 392$,
  $h^*(V) = (0, 120, 0, 0)$ (no. of **27**'s)
- Here $||G_1|| = (2, 4)$, $||G_2|| = (2, 5)$, $||F_1^1|| = (2, 8)$, $||F_2^1|| = (3, 7)$,
  $||F_3^1|| = (3, 7)$, $||F_4^1|| = (1, 2)$
- Sum of third and fourth F equals sum of two hypersurface degrees.
- Redefine: $\tilde{\Gamma}^1 = \langle p_1 \rangle \Lambda^3$, $\tilde{\Gamma}^2 = \langle p_1 \rangle \Lambda^4$, $\tilde{\Lambda}^3 = \frac{\Gamma^1}{\langle p_1 \rangle}$, $\tilde{\Lambda}^4 = \frac{\Gamma^2}{\langle p_1 \rangle}$, $\tilde{G} = F_3^1$,
  $\tilde{G}_2 = F_4^1$, $\tilde{F}_3^1 = G_1$, $\tilde{F}_4^1 = G_2$
- Superpotential: $\mathcal{W} = \tilde{\Gamma}^1 \tilde{G}_1 + \tilde{\Gamma}^2 \tilde{G}_2 + \langle p_1 \rangle (\tilde{\Lambda}^3 \tilde{F}_3^1 + \tilde{\Lambda}^4 \tilde{F}_4^1 + \Lambda^1 F_1^1 + \Lambda^2 F_2^1)$

Example

- $||\tilde{G}_1|| = (3,7)$, $||\tilde{G}_2|| = (1,2)$, $||\tilde{F}^1_3|| = (2,4)$, $||F^1_4|| = (2,5)$, $||\tilde{\Gamma}^1|| = (-3,-7)$, $||\tilde{\Gamma}^2|| = (-1,-2)$ $||\tilde{\Lambda}^3|| = (1,4)$, $||\tilde{\Lambda}^4|| = (1,3)$.
- Leads to new geometry $(\widetilde{X}, \widetilde{V})$

x_i						Γ^j	Λ^a				p_l
0	0	0	1	1	1	-3	1	0	1	1	-3
1	1	1	2	2	0	-7	0	1	4	3	-8

- $dim(\widetilde{\mathcal{M}}_0) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 2 + 95 + 295 = 392$, $h^*(\widetilde{V}) = (0, 120, 0, 0)$
- Here $h^{1,1}$ stays fixed, complex structure and bundle moduli interchange.
- More general mixing possible...e.g. increase # of $U(1)$s $\rightarrow$ can mix all moduli

The question...how are "dual" theories related?

- In 2011, Blumenhagen + Rahn performed a landscape scan. Tested duality by counting states:

$$h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V}))$$

  and charged matter in $\sim 80,000$ examples. Agreement in nearly all cases.
- Question: Can duality be tested (even in the geometric, perturbative regime) in more detail?
- Recall, these are $N = 1$ 4D theories. Want more than $dim(\mathcal{M}_0)$... $\Rightarrow$ **Moduli can be obstructed.**
- Can we compare the effective potential and vacuum space of the chain of dual theories?
- Must engineer examples with interesting/calculable potentials...

Engineering potentials...

- Conditions for $N = 1$ Supersymmetry in $4D$: Hermitian-Yang Mills Eqns
  $F_{ab} = F_{\bar{a}\bar{b}} = 0, g^{a\bar{b}} F_{\bar{b}a} = 0$
- $g^{a\bar{b}} F_{\bar{b}a} = 0 \Leftrightarrow$ Donaldson-Uhlenbeck-Yau Thm: V is slope (Mumford) poly-stable.
  - Slope: $\mu(V) \equiv \frac{1}{\mathrm{rk}(V)} \int_X c_1(V) \wedge \omega \wedge \omega$ where $\omega = t^k \omega_k$ is the Kahler form on X (ω_k a basis for $H^{1,1}(X)$).
  - V is Stable if for every sub-sheaf, $\mathcal{F} \subset V$, with $0 < rk(\mathcal{F}) < rk(V)$,
    $$\mu(\mathcal{F}) < \mu(V)$$
  - V is Poly-stable if $V = \bigoplus_i V_i$, V_i stable such that $\mu(V) = \mu(V_i)$ $\forall i$
- $F_{ab} = F_{\bar{a}\bar{b}} = 0$: V is holomorphic.
- Stability $\Leftrightarrow$ $4D$ D-terms
- Holomorphy $\Leftrightarrow$ $4D$ F-terms

The idea

(Work w/ H. Feng):

- Create bundles that are only stable or holomorphic for sub-loci in moduli space $\Rightarrow$ what happens under TSD?
- E.g. Strictly poly-stable bundle $\Rightarrow$ 4D D-terms restrict Kähler moduli
- E.g. Bundle holomorphic only on a sub-locus in the complex structure moduli space of X $\Rightarrow$ 4D F-terms obstruct complex structure moduli.

D-term Example

x_i						Γ^j	Λ^a								p_l		
1	1	0	0	0	0	−2	1	−1	0	0	2	1	1	2	−3	−1	−2
0	0	1	1	1	1	−4	−1	1	1	1	1	2	2	2	−2	−4	−3

with initial total moduli count

$$dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(X, End_0(V)) = 2 + 86 + 340 = 428$$

- Naively: $rk(V) = 5$, $c_1(V) = 0 \Rightarrow SU(5)$ 4D theory.
- In fact, V stable *only* on a ray in Kähler moduli space $t^2 = 4t^1$.
- Only supersymmetric configuration of V: $V \to U_3 \oplus L \oplus L^\vee$ w/ $L = \mathcal{O}(1, -1)$.

Features of interest

- Non-trivial D-term lifts one Kähler modulus. Reduction in moduli $dim(\mathcal{M}_1) = dim(\mathcal{M}_0) - 1 = 427$
- Non-Abelian Enhancement: Structure group is not $SU(5)$, rather $S[U(1) \times U(1)] \times SU(3) \subset E_8 \Rightarrow SU(6) \times U(1)$, with $U(1)$ symmetry Green-Schwarz massive.
- Bundle forced to locus w/ non-Abelian symmetry enhancement
- From the stability wall, can explore branch structure into nearby geometries $\Rightarrow \langle C_{bundle} \rangle \neq 0 \rightarrow$ deform to irreducible bundle. Higgsing $SU(6) \times U(1) \rightarrow SU(5)$
- How much of this is visible in the TS duals?
- In this case can construct a chain of 17 TS dual geometries w/ conifold-type "splits" $\rightarrow h^{1,1} + 1$. What do we get?...

One example of TSD dual:

x_i								Γ^j		Λ^a								p_l		
0	0	0	0	0	0	1	1	-1	-1	0	0	1	0	0	0	0	0	0	0	-1
1	1	0	0	0	0	0	0	-2	0	1	-1	0	0	2	1	1	2	-3	-1	-2
0	0	1	1	1	1	0	0	-2	-2	-1	1	-1	1	3	2	2	2	-2	-4	-3

Questions:

1. Does $(\widetilde{X}, \widetilde{V})$ give rise to a stability wall? ✓
2. Here $dim(\widetilde{\mathcal{M}}_0) = 429$? But $dim(\widetilde{\mathcal{M}}_1) = 427$ ✓
3. Do the structure group and charged matter spectra of the two theories match? ✓
4. Does the vacuum branch structure (i.e. local deformation space) correspond? ✓... In general, find commutative diagram:

$$\begin{array}{ccccc} & V_1 & \xrightarrow{dual} & \widetilde{V}_1 & \\ \langle C \rangle & \downarrow & & \downarrow & \langle \tilde{C} \rangle \\ & V_2 & \xrightarrow{dual} & \widetilde{V}_2 & \end{array}$$

Holomorphic Vector bundles

- V holomorphic if $F_{ab} = F_{\bar{a}\bar{b}} = 0$
- Suppose we begin with a holomorphic bundle and then vary the complex structure? Must a bundle stay holomorphic for any variation $\delta \mathfrak{z}^I v_I \in h^{2,1}(X)$? $\Rightarrow$ No: Need $\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)y}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A^y_{\bar{b}]} = 0$
- $0 \to V \otimes V^{\vee} \to \mathcal{Q} \stackrel{q}{\to} TX \to 0$ is known as the Atiyah sequence.
- The long exact sequence in cohomology gives us

$$0 \to H^1(V \otimes V^{\vee}) \to H^1(\mathcal{Q}) \stackrel{dq}{\to} H^1(TX) \stackrel{\alpha}{\to} H^2(V \otimes V^{\vee}) \to \dots$$

- If the map dq is surjective then $H^1(\mathcal{Q}) = H^1(V \otimes V^{\vee}) \oplus H^1(TX)$
- But dq not surjective in general! $H^1(\mathcal{Q}) = H^1(V \otimes V^{\vee}) \oplus Im(dq)$
- dq difficult to define, but by exactness, $Im(dq) = Ker(\alpha)$ where $\alpha = [F^{1,1}] \in H^1(V \otimes V^{\vee} \otimes TX^{\vee})$ is the Atiyah Class

Deformation Theory

There are three objects in deformation theory that we need

- $Def(X)$: Deformations of X as a complex manifold. Infinitesimal defs parameterized by the vector space $H^1(TX) = H^{2,1}(X)$. These are the *complex structure* deformations of X.
- $Def(V)$: The deformation space of V (changes in connection, δA) *for fixed* C.S. moduli. Infinitesimal defs measured by $H^1(End(V)) = H^1(V \otimes V^\vee)$. These define the *bundle moduli* of V.
- $Def(V, X)$: Simultaneous holomorphic deformations of V and X. The tangent space is $H^1(X, \mathcal{Q})$ where

$$0 \to V \otimes V^\vee \to \mathcal{Q} \xrightarrow{\pi} TX \to 0$$

If $\mathcal{Z}$ is the (projectivized) total space of the bundle, $\mathcal{Q} = r_* T\mathcal{Z}$ (Donaldson) and $r : \mathcal{Z} \to X$.

GVW Superpotential and F-terms

- For the $4D$ Theory: Gukov-Vafa-Witten superpotential $W = \int_X \Omega \wedge H$ where $H = dB - \frac{3\alpha'}{\sqrt{2}} \left(\omega^{3\text{YM}} - \omega^{3\text{L}}\right)$
- In Minkowski vacuum (with $W = 0$), F-terms: $F_{C_i} = \frac{\partial W}{\partial C_i} = -\frac{3\alpha'}{\sqrt{2}} \int_X \Omega \wedge \frac{\partial \omega^{3\text{YM}}}{\partial C_i}$
- Dimensional Reduction Anzatz: $A_\mu = A_\mu^{(0)} + \delta A_\mu + \bar{\omega}_\mu^i \delta C_i + \omega_\mu^i \delta \bar{C}_i$

$$F_{C_i} = \int_X \epsilon^{\bar{a}\bar{c}\bar{b}} \epsilon^{abc} \Omega^{(0)}_{abc} 2\bar{\omega}^{xi}_{\bar{c}} \,\text{tr}(T_x T_y) \left(\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)y}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A^y_{\bar{b}]}\right)$$

- Computationally same as Atiyah obstructions.
- Superpotential observations in lit. since 80's. Hard part is engineering calculable examples.
- Idea: Build bundles where "ingredients" crucially depend on complex structure...

F-term Example

- What about TSD e.g.s w/ holomorphy obstructions? (i.e. F-term lifting)?
- Consider the $SU(2)$ bundle

x_i						Γ^j	Λ^a			p_l
1	1	0	0	0	0	-2	2	-1	-1	0
0	0	1	1	1	1	-4	0	2	2	-4

- Does not define a stable bundle for general choices of complex structure: Missing a map $F_1^a \in H^0(X, \mathcal{O}(-2,4) = 0$ generically. However, line bundle cohomology can jump...
- Shown in (arXiv:1107.5076) that on a 53-dim. sublocus of CS moduli space, $h^0(X, \mathcal{O}(-2,4) = 1$. $\Rightarrow dim(\mathcal{M}_1) = dim(\mathcal{M}_0) - 33$.
- TSD matches $dim(\mathcal{M}_1)$ exactly, but for the remainder of this talk, I want to take step back and look at geometry of (X, V)...

$(0, 2)$ GLSMs as a geometric playground

- Question: In case of TSD or other heterotic "redundancies", can we produce Donaldson's (projective) total spaces of the bundle and can compare their properties?
- Hope is to extract "essential" features of manifold/bundle pair.
- Questions at both the level of the metric (i.e. differential geometry) and topology/algebraic description (i.e. algebraic geometry)

Inspired by GLSMs, let's begin by considering the case of X a CY complete intersection manifold in a toric variety and V defined via a monad

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r_{\mathcal{V}}} \xrightarrow{\otimes E_i^a} \oplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a^l} \oplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \to 0$$

with $V = \frac{ker(F_a^l)}{im(E_i^a)}$

- Result:

  Let V be a stable, holomorphic $SU(n)$ bundle.

  V is defined via a monad over a toric CICY 3-fold iff its projectivized total space, $\mathcal{Z} = \mathbb{P}(V \to X)$ is an $dim_{\mathbb{C}} = 3 + (n-1)$ (Kahler) toric complete intersection manifold.

Illustrative example:

- Given $X = \mathbb{P}^5[2,4]$ with $0 \to V \to \mathcal{O}(1)^{\oplus 7} \to \mathcal{O}(3) \oplus \mathcal{O}(2)^{\oplus 2} \to 0$
- $dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 1 + 89 + 159 = 249$
- $\mathcal{Z}$ defined by

$$\mathcal{Z} = \left[\begin{array}{c|ccccc} \mathbb{P}^6 & 0 & 0 & 1 & 1 & 1 \\ \mathbb{P}^5 & 2 & 4 & 2 & 1 & 1 \end{array} \right]$$

  $\mathcal{Z}$ is a Kähler 6-fold with $h^{1,1} = 2$ and $h^1(T\mathcal{Z}) = 248$.
- Neat feature: The ambient space is determined by the ambient spaces of bundle/monad. If X is a CICY in $\mathcal{A}$ and if $0 \to V \to B \to C \to 0$ is a monad, can define fiber space of V as CI in $\mathcal{E} = \mathbb{P}(\pi : B \to X)$ (total ambient space not in general a product).

- Total space $\Leftrightarrow (X, V)$?

$$0 \quad \to \quad \mathcal{O}^{\oplus r} \to \bigoplus_{i=1} \mathcal{O}(\mathbf{D}_i) \to T\mathcal{A} \to 0$$
$$0 \to T\mathcal{Z} \to T\mathcal{A} \to \mathcal{N} \to 0$$

with $\mathbf{D_i}$ determined by GLSM charges and the $\mathcal{N} = \bigoplus_{j=1} \mathcal{O}(\mathbf{P}_j)$, with $\mathbf{P}_j$ is the multi-degree of the j-th hypersurface.

- How to construct X, V? If $0 \to A \xrightarrow{E} B \xrightarrow{F} C \to 0$ is a three-term monad, the "display" is useful:

$$\begin{array}{ccccccccc} & & 0 & & 0 & & 0 & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & A & \to & K & \to & V & \to & 0 \\ & & || & & \downarrow & & \downarrow & & \\ 0 & \to & A & \to & B & \to & Q & \to & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & 0 & \to & C & = & C & \to & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & 0 & & 0 & & 0 & & \end{array}$$

where $K = \ker(F)$ and $Q = \mathrm{coker}(E)$.

To reconstruct (X, V) from $\mathcal{Z}$, consider the display and restrict to fiber and base. E.g.

$$
\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \rightarrow & \mathcal{O} & \rightarrow & \pi^*(V) \otimes \xi_{\mathcal{Z}} & \rightarrow & T_{\mathcal{Z}|X} & \rightarrow & 0 \\
 & & || & & \downarrow & & \downarrow & & \\
0 & \rightarrow & \mathcal{O} & \rightarrow & \pi^*(B) \otimes \xi_{\mathcal{Z}} & \rightarrow & T_{\mathcal{E}|X}|_{\mathcal{Z}} & \rightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \rightarrow & 0 & \rightarrow & \pi^*(C) \otimes \xi_{\mathcal{Z}} & = & \mathcal{N}|_{\mathcal{Z}} & \rightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}
$$

We have "reconstructed" V from the $\mathbb{P}^{n-1}$ fiber of $\mathcal{Z}$.

Useful for systematically classifying heterotic geometries?

It is natural to ask where the degrees of freedom of the heterotic theory are realized in $\mathcal{Z}$?

- $h^1(\mathcal{Z}, T\mathcal{Z}) = h^1(X, \mathcal{Q})$ (i.e. the complex moduli of a heterotic theory)
- $h^{1,1}(\mathcal{Z}) = 1 + h^{1,1}(X)$ (one more than the number of Kahler moduli. Dilaton?)
- The *tautological line bundle* $\xi_{\mathcal{Z}}$ is uniquely defined by the properties that $\xi_{\mathcal{Z}}|_F = \mathcal{O}_F(1)$ and $\pi_*(\xi_{\mathcal{Z}}) = V$ (moreover $c_1(\mathcal{Z}) = n\xi_{\mathcal{Z}}$ for an $Su(n)$ bundle). For cases of interest,

$$h^*(Z, \xi_{\mathcal{Z}}) = h^*(X, V) \text{ i.e. counts charged matter}$$

- Chern classes: $ch(\mathcal{Z}) = function(ch_2(X) = ch_2(V), ch_3(V))$, etc

Summary

- $(0,2)$ Target Space Duality leads to a wealth of intriguing geometric correspondences. Worthy of further study...
- We have begun a systematic rewriting of heterotic geometry in terms of the total space of the bundle
- In the case of $SU(n)$ monad bundles and toric CICY 3-folds $\Rightarrow$ explicit realization of Kähler $(n+2)$-fold as a toric CICY.
- In the simplest cases of $(0,2)$ "dualities", Hodge numbers, Chern classes and cohomology of the tautological line bundle are identical $\Rightarrow$ Sigma model automorphism?
- In other dualities, can in principle track geometric transitions in total space $(1 \leftrightarrow 1?)$, properties of 4D EFT preserved.

Summary

- Utility for new $(0,2)$ dualities? String Pheno? Systematic constructions?
- Further study underway...