FROM ASYMPTOTICS TO EXACT RESULTS IN STRING AND GAUGE THEORIES

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$$\sum_{n=0}^{\infty} E_n\, g^n \quad \mathrm{e}^{-A/g}$$

PERTURBATION THEORY

Perturbation theory: fundamental in computation of observables

- Ground state energies in quantum mechanical problems
- Beta-functions in quantum field theories
- Genus expansion of string theory
- Large-N expansions in gauge theories
- Late-time behaviour of strongly correlated systems

…

BUT: expansions are often divergent, ***asymptotic***!

ASYMPTOTIC EXPANSIONS

Perturbation theory often leads to divergent asymptotic expansions

Surprisingly, this asymptotic behaviour carries crucial information about exponentially small, ***non-perturbative (NP) phenomena governing the global analytic properties*** of physical observables

Goal: Global Analytic properties of observables beyond perturbation theory

In this talk: Focus on late-time behaviour of the energy density of strongly coupled plasma

OUTLINE

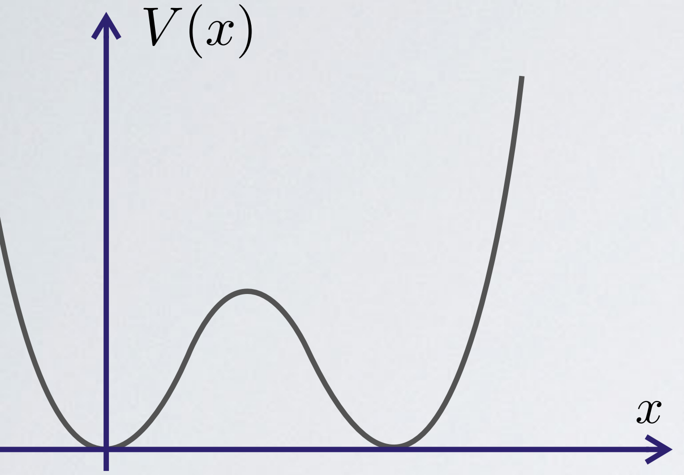
1. A primer on resurgent transseries (?)
2. Late-time behaviour for strongly coupled plasma
   - Microscopic description and dual gravity solution
   - Asymptotic analysis and QNMs
3. A simpler case: Müller-Israel-Stuart hydrodynamics
   - Transseries, asymptotics and summation prescriptions
   - The attractor solution from asymptotic late-times?
4. Further applications

1.
A PRIMER ON RESURGENT TRANSSERIES (?)

[IA,Basar,Schiappa'18]

PERTURBATION THEORY IN QM

g very small $\xrightarrow[\text{theory}]{\text{Perturbation}}$ $\boxed{E_{g.s.}(g) \simeq \sum_{n=0}^{\infty} E_n\, g^n}$



$$V(x) = \frac{1}{2} x^2 \left(1 - \sqrt{g}\, x\right)^2$$

- ***Series is asymptotic***: For large enough n

$$E_n \sim n!\, A^{-n}$$

Why asymptotic? Existence of instantons

Corrections to $E_{g.s.} \sim \mathrm{e}^{-A/g} \sum_{n=0}^{\infty} E_n^{(1)}\, g^n$ Suppressed!

BEYOND PERTURBATION THEORY

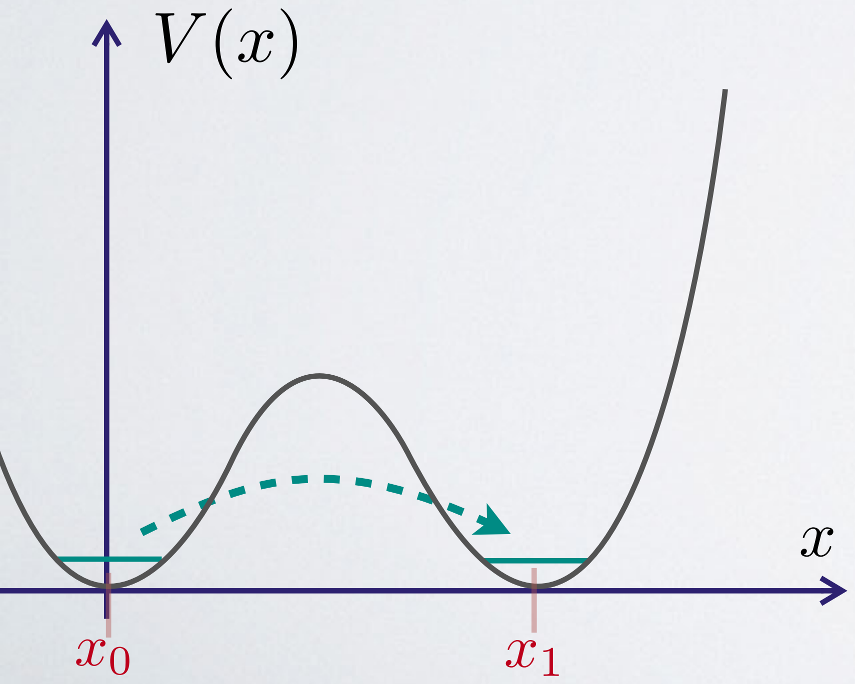
g very small → $E_{g.s.}(g) \simeq \sum_{n=0}^{\infty} E_n^{(0)} g^n$

$+$

Instanton corrections to $E_{g.s.}$ → $\sim \mathrm{e}^{-A/g} \sum_{n=0}^{\infty} E_n^{(1)} g^n$

$+$

Higher instanton corrections → $\mathcal{O}\left(\mathrm{e}^{-2A/g}\right)$



[Vanstein'64;Bender,Wu'73;Bogomolny,Zinn-Justin'80]

TRANSSERIES SOLUTION

$$E_{g.s.}(g,\sigma) \simeq \sum_{k=0}^{\infty} \sigma^k \mathrm{e}^{-kA/g} E^{(k)}(g) \qquad E^{(k)}(g) \simeq \sum_{n=0}^{\infty} E_n^{(k)}\, g^n$$

k-instanton contribution,
each is asymptotic

$$E_n^{(k)} \sim n!\,(kA)^{-n}$$

Formal expansion in transmonomials

- the small parameter g
- non-perturbative term $\mathrm{e}^{-A/g}$
- σ encodes boundary/initial conditions

$E_{g.s.}(g,\sigma)$ requires all instantons to be well defined

[Edgar'08]

RESURGENCE

$$E^{(k)} \sim \sum_{n=0}^{\infty} E_n^{(k)} \, g^n$$

Coefficients between different sectors are related through large-order relations

Look at perturbative coefficients for large enough n

$$E_n^{(0)} \sim \frac{n!}{A^n}\left(E_1^{(1)} + \frac{A}{n-1}E_2^{(1)} + \cdots\right) + \frac{n!}{(2A)^n}\left(E_1^{(2)} + \frac{2A}{n-1}E_2^{(2)} + \cdots\right) + \cdots$$

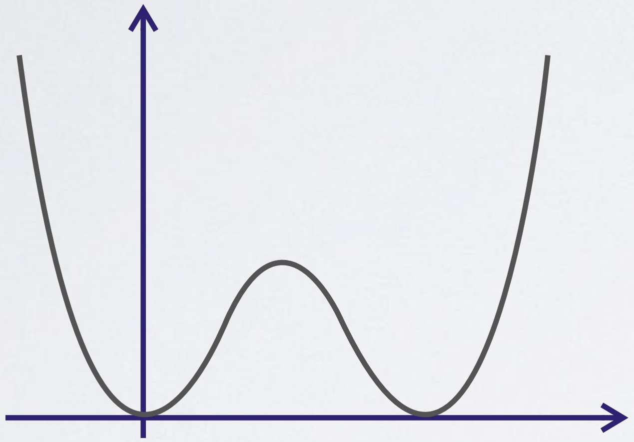
Same is true for all instanton coefficients

Using Resurgence

large order relations encode NP information in the perturbative series

BOREL TRANSFORMS

Determine NP phenomena from an asymptotic series



$$E_{g.s.}(g) \simeq \sum_{n=0}^{\infty} E_n^{(0)} g^n$$

$$E_n^{(0)} \sim \frac{n!}{A^n}$$ for large enough n

Remove the factorial growth to get a convergent series: inverse Laplace transform to each term

$$B_E(s) = \sum_{n=0}^{\infty} \frac{E_n^{(0)}}{n!} s^n$$

BOREL TRANSFORM

Let's simplify our example!

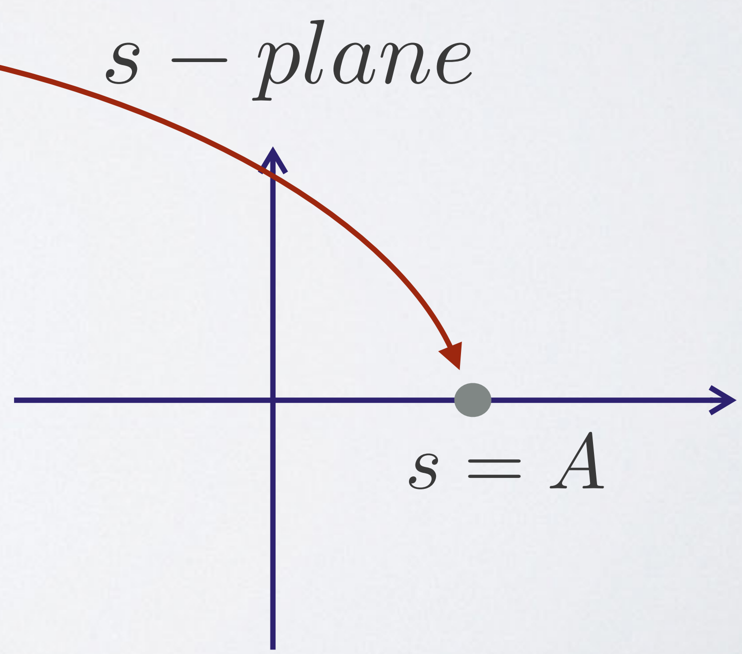
Original Series:

$$E_{g.s.}(g) \simeq \sum_{n=0}^{\infty} E_n^{(0)}\, g^n$$

$$E_n^{(0)} = \frac{n!}{A^n}$$

Borel transform:

$$B_E(s) = \sum_{n=0}^{\infty} \frac{E_n^{(0)}}{n!} s^n = \sum_{n=0}^{\infty} \left(\frac{s}{A}\right)^n = \frac{A}{A-s}$$

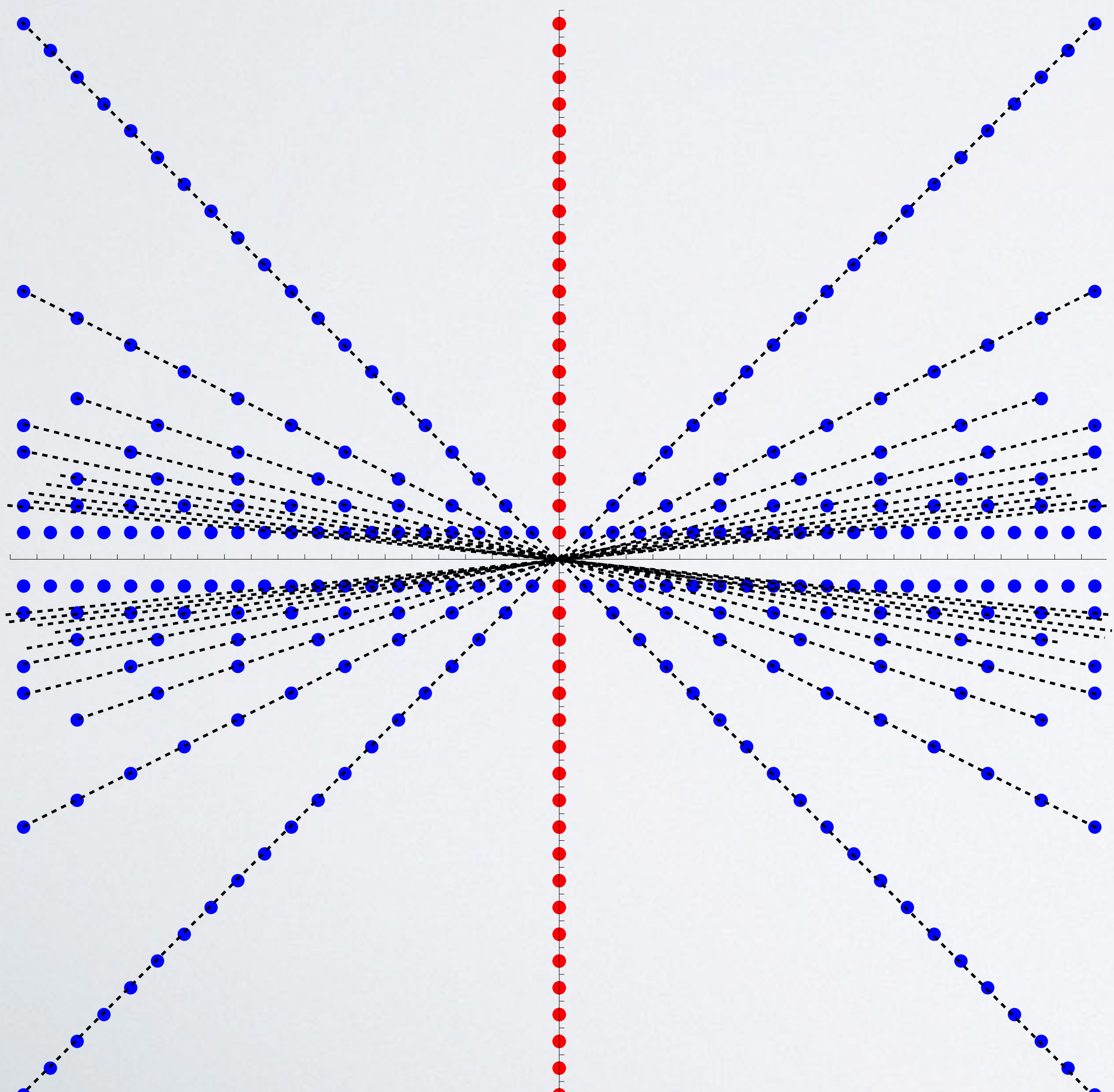


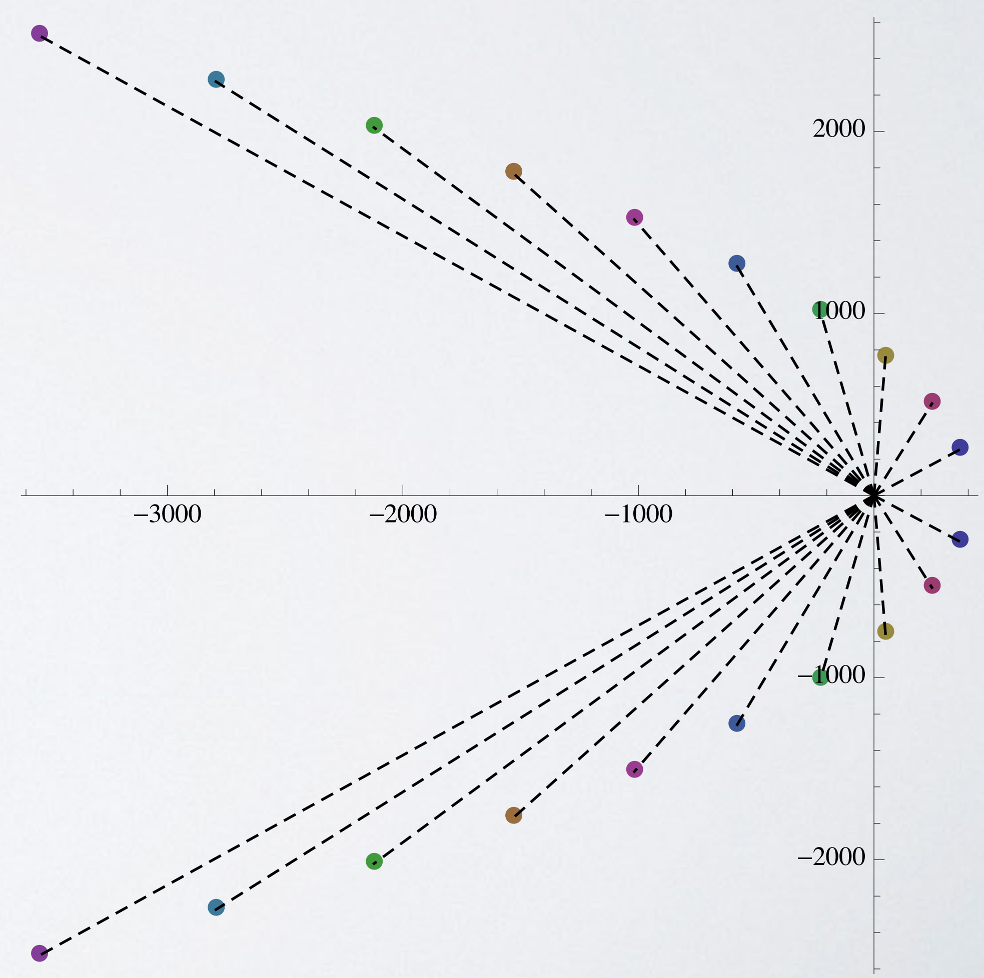
single pole at $s = A$

The position of the pole is controlled by instanton action

BOREL TRANSFORM

- Non-perturbative phenomena: singularities in Borel plane
- Singularities usually will be branch cuts
- Singular directions: ***Stokes lines***
- Structure of singularities can be very complex





[IA,Russo,Schiappa'14]

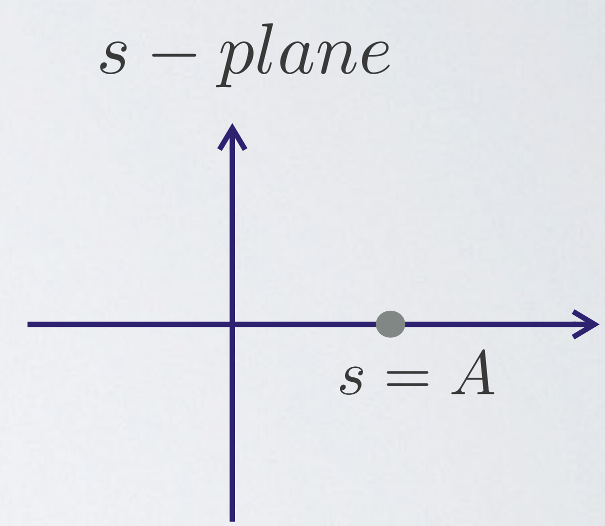
BOREL RESUMMATION

Original Series:

$$E_{g.s.}(g) \simeq \sum_{n=0}^{\infty} \frac{n!}{A^n} g^n$$

Borel Transform:

$$B_E(s) = \sum_{n=0}^{\infty} \left(\frac{s}{A}\right)^n = \frac{A}{A-s}$$



How to associate a function to the original asymptotic series?

Via ***Borel resummation***: Laplace transform

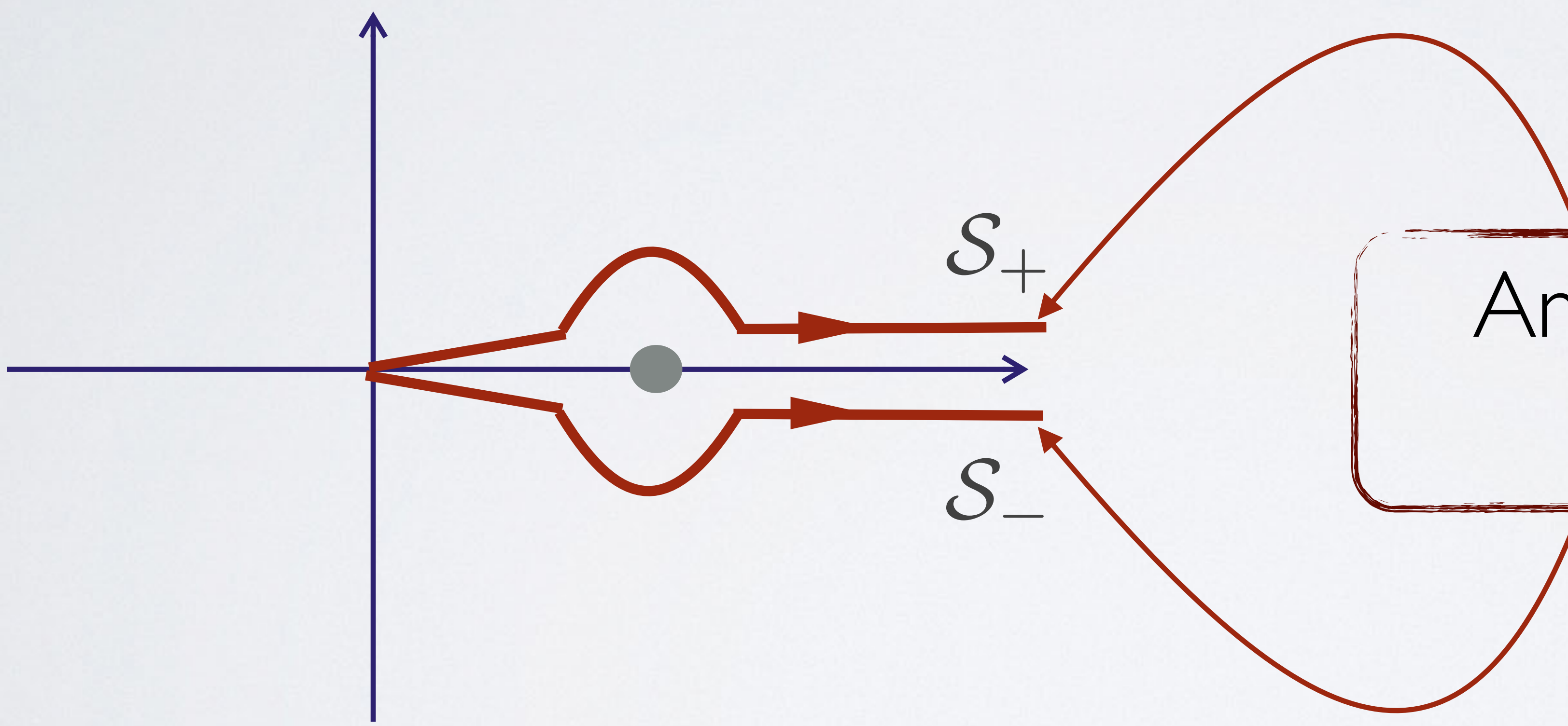
$$\mathcal{S}E_{g.s.}(g) = \int_0^{\infty} \mathrm{d}s B_E(s) \mathrm{e}^{-s/g}$$

BOREL RESUMMATION

$$\mathcal{S}E_{g.s.}(g) = \int_0^\infty \mathrm{d}s B_E(s) \mathrm{e}^{-s/g}$$

$$\boxed{B_E(s) = \frac{A}{A-s}}$$

How do we integrate with along a singular direction?



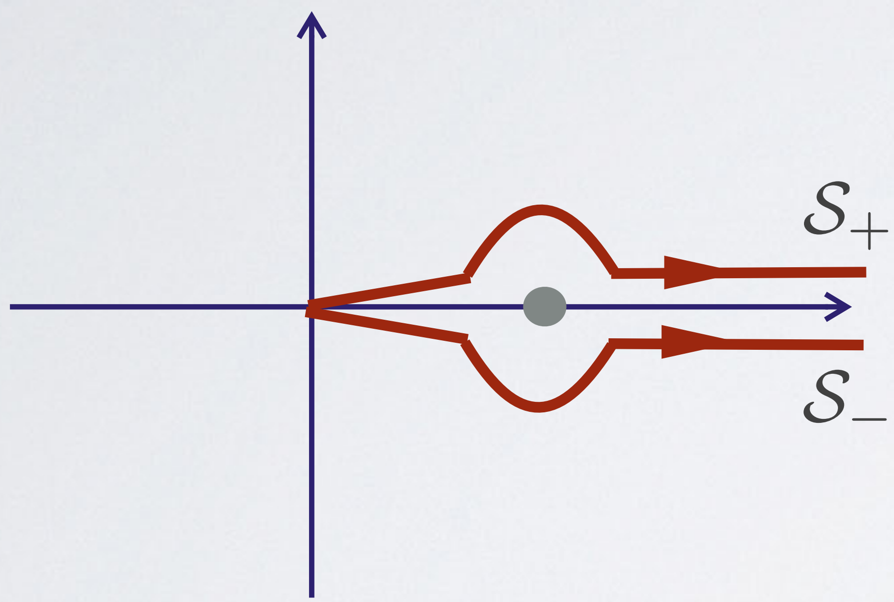
Ambiguity in choice of contour

$$\mathcal{S}_+ - \mathcal{S}_- = \oint_{s=A} \mathrm{d}s \frac{A}{A-s} \mathrm{e}^{-s/g} \sim \mathrm{e}^{-A/g}$$

Non-perturbative Ambiguity

BOREL RESUMMATION

- Borel resummation straightforward in the directions without singularities
- Re-summation along Stokes directions: ambiguities



$$\mathcal{S}_+ E_{g.s.}(g,\sigma) = \mathcal{S}_- E_{g.s.}(g,\sigma + S)$$

Stokes constant (imaginary)

Ambiguities in the transseries

- all sectors have ambiguities
- Use resurgence to fix σ s.t.

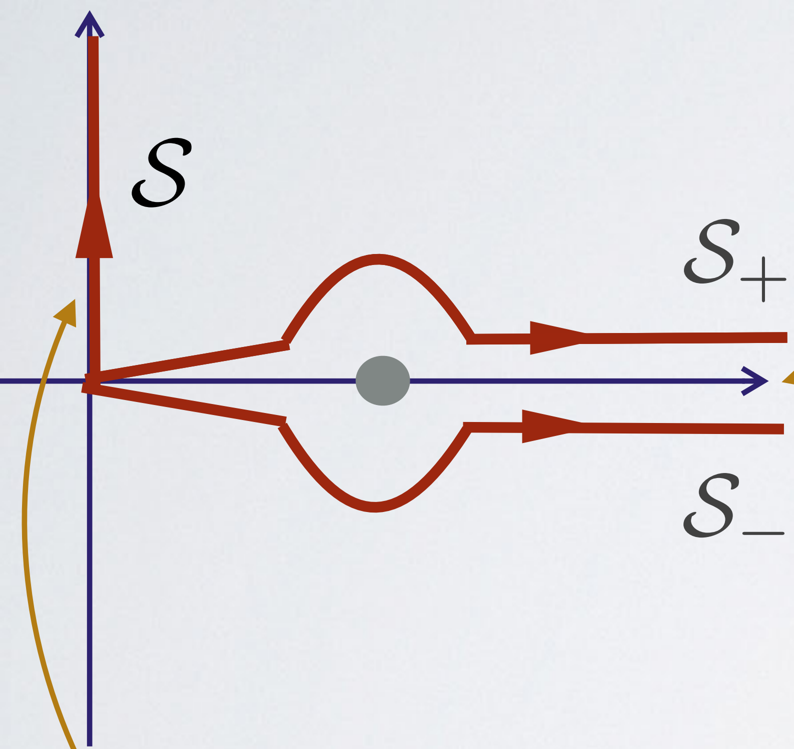
$$(\mathcal{S}_+ - \mathcal{S}_-)\, E_{g.s.}(g,\sigma_0) = 0$$

[Delabaere'99][IA,Schiappa'13]

The full transseries is unambiguous, and we can construct an analytic solution in **any** direction

STOKES PHENOMENA

Transseries $E_{g.s.} \simeq \sum_{k=0}^{\infty} \sigma^k e^{-kA/g} E^{(k)}(g)$ $E^{(k)} \sim \sum_{n=0}^{\infty} E_n^{(k)} g^n$



Across Stokes line

$\mathbb{I}\mathrm{m}(A/g) = 0$ Non-pert contributions very suppressed

$$\mathcal{S}_+ E_{g.s.}(g,\sigma) = \mathcal{S}_- E_{g.s.}(g,\sigma + S)$$

At anti-Stokes line

$\mathbb{R}\mathrm{e}(A/g) = 0$ All terms of same order! **Different physics**

IN SUMMARY

- Obtain NP completion in the form of transseries

$$E_{g.s.} \simeq \sum_{k=0}^{\infty} \sigma^k \mathrm{e}^{-kA/g}\, E^{(k)}(g) \qquad\qquad E^{(k)} \sim \sum_{n=0}^{\infty} E_n^{(k)}\, g^n$$

- Re-sum all asymptotic sectors $\mathcal{S}_+ E^{(k)}(g)$

- Determine σ from external data (boundary/initial conditions)

- This can be done for any value of g and encodes:

  Analytic data (poles, zeros, branch cuts)

  Phase transitions (Stokes phenomena)

2.

LATE-TIME ASYMPTOTIC FOR STRONGLY COUPLED PLASMA IN $\mathcal{N} = 4$ SYM

RELATIVISTIC HYDRODYNAMICS

It provides a reliable description of strongly coupled systems

- real life: strongly coupled quark-gluon plasma in particle accelerators;
- To determine the kinetic parameters of hydrodynamic equations (e.g. shear viscosity): study the associated microscopic theory

The associated microscopic theory can be a QFT, such as strongly coupled $\mathcal{N}=4$ Super Yang-Mills (SYM)

$N \to \infty$ ***gauge/gravity duality:*** determine hydrodynamic parameters, time dependent processes of the SYM plasma from dual geometry

[Policastro *et al* '01-'04; Nastase '05]

STRONGLY COUPLED SYSTEMS

Kinematic regime: ***expanding plasma*** in the so-called central rapidity region, where one assumes ***longitudinal boost invariance*** (Bjorken flow)

[Bjorken '83]

In hydrodynamic theories the energy-momentum tensor is given by

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}(\mathcal{E})(\eta^{\mu\nu} + u^\mu u^\nu) + \Pi^{\mu\nu}$$

Energy density

Pressure, in 4d conformal theories given by:

$$\mathcal{P}(\mathcal{E}) = \mathcal{E}/3$$

flow velocity

Shear stress tensor: dissipative effects

Symmetries: conformal invariance, transversely homogeneous, invariance under longitudinal Lorentz boosts

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Strongly coupled SYM boost invariant plasma:
all physics encoded in $\mathcal{E}(\tau)$.
Obtaining this function is in general too difficult:
perform a ***large proper time expansion*** $\tau \gg 1$ **.**

LATE TIME BEHAVIOUR

Starting from highly non-equilibrium initial conditions, the microscopic theory will reveal the transition to hydrodynamic behaviour at late times

Conformal theories: late-time behaviour of energy density highly constrained

$$\mathcal{E}(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}}\left(1 + \sum_{k=1}^{+\infty} \frac{\epsilon_k}{(\Lambda\tau)^{2k/3}}\right), \quad \tau \gg 1$$

- Λ is a dimensionful parameter encoding initial non-eq. conditions
- Leading behaviour predicted by boost-invariant perfect fluid
- Subleading terms: dissipative hydrodynamic effects

Next: use dual geometry to analyse the expansion of boost invariant SYM plasma

SYM PLASMA FROM ADS/CFT

Equilibrium states of the microscopic theory (CFT)	$\longleftrightarrow$	black hole solutions [Witten '98]
flat space at boundary: planar horizons	$\longleftrightarrow$	black branes
Perturbative non-equilibrium phenomena	$\longleftrightarrow$	linearised perturbations of black brane solution
Non-hydrodynamic d.o.f.	$\longleftrightarrow$	exp. decaying black branes' quasi-normal modes

[Janik, Peschanski '05][Janik '05]

SYM PLASMA FROM ADS/CFT

Dual geometry given by boost invariant 5D metric

$$ds^2 = \frac{1}{z^2}\left(dz^2 - \mathrm{e}^{-A}d\tau^2 + \tau^2\mathrm{e}^{B}dy^2 + \mathrm{e}^{C}d\boldsymbol{x}_\perp^2\right) = \frac{1}{z^2}\left(G_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

Solve Einstein equations with negative cosmological constant
(asymptotic behaviour is AdS)

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R - 6G_{\mu\nu} = 0$$

- metric components depend on $z,\ \tau$
- boundary condition at $z = 0$:

$$G_{\mu\nu} = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + \cdots$$

Energy density $\mathcal{E}\left(\tau\right) = -\lim_{z\to 0}\frac{A\left(z,\tau\right)}{z^4}$

[Hare *et al* '00][Skenderis '02][Fefferman,Graham '85]

SYM PLASMA FROM ADS/CFT

Metric ansatz: multi-parameter transseries with exponential decaying sectors and perturbative expansions in proper time

The most general solution for the ***energy density*** of the SYM plasma is:

$$\mathcal{E}\left(u \equiv \tau^{2/3}, \boldsymbol{\sigma}\right) = \sum_{\boldsymbol{n} \in \mathbb{N}_0^\infty} \boldsymbol{\sigma}^{\boldsymbol{n}} \mathrm{e}^{-\boldsymbol{n}\cdot\boldsymbol{A}\,u} \Phi_{\boldsymbol{n}}(u) \;, \quad \Phi_{\boldsymbol{n}}(u) = u^{-\beta_{\boldsymbol{n}}} \sum_{k=0}^{+\infty} \varepsilon_k^{(\boldsymbol{n})} u^{-k}$$

exponentially decaying coupled QNMs $\omega_k = -\frac{2\mathrm{i}}{3} A_k$

perturbative late-time expansions

- Infinite number of QNMs $\boldsymbol{A} = (A_1, \bar{A}_1, A_2, \bar{A}_2, \cdots)$
- Parameters encoding non-hydro initial conditions $\boldsymbol{\sigma} = (\sigma_{A_1}, \sigma_{\bar{A}_1}, \sigma_{A_2}, \sigma_{\bar{A}_2}, \cdots)$

All expansions in the energy density are ***asymptotic***!

[Heller,Janik,Witaszczyk'15; IA *et al*'18]

ASYMPTOTIC ENERGY DENSITY

Hydrodynamic expansion: $$\Phi_0\left(u\right) = u^{-2}\sum_{k=0}^{+\infty} \varepsilon_k^{(0)}\, u^{-k} \qquad \varepsilon_k^{(0)} \sim \frac{k!}{|A_1|}$$

Singularities in Borel plane:

ω_1; $2\omega_1$; $3\omega_1$

ω_2

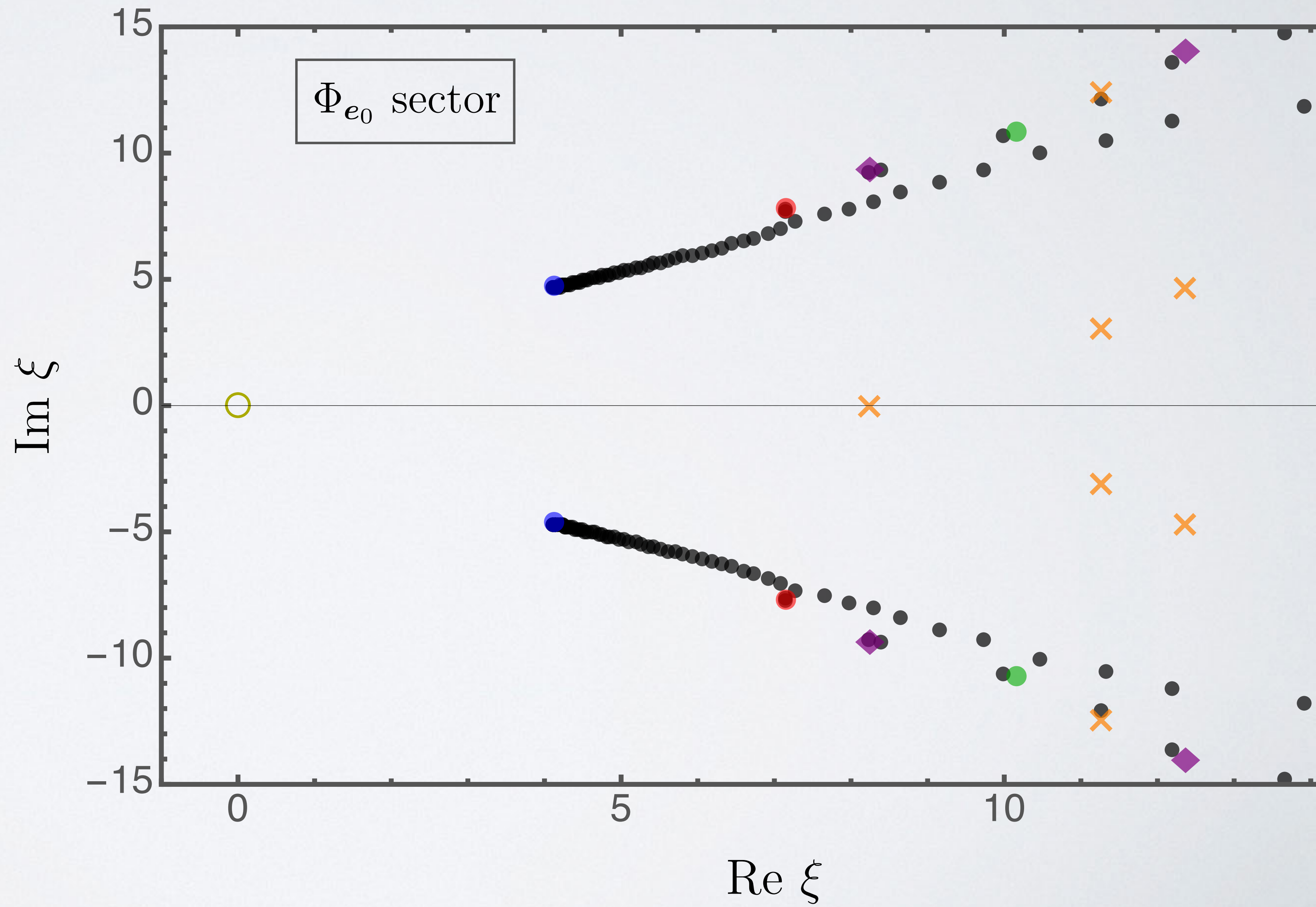
ω_3

$\overline{\omega}_i$

$$\omega_1 = \frac{3}{2}(2.746676 + 3.119452\mathrm{i})$$

$$\omega_2 = \frac{3}{2}(4.763570 + 5.169521\mathrm{i})$$

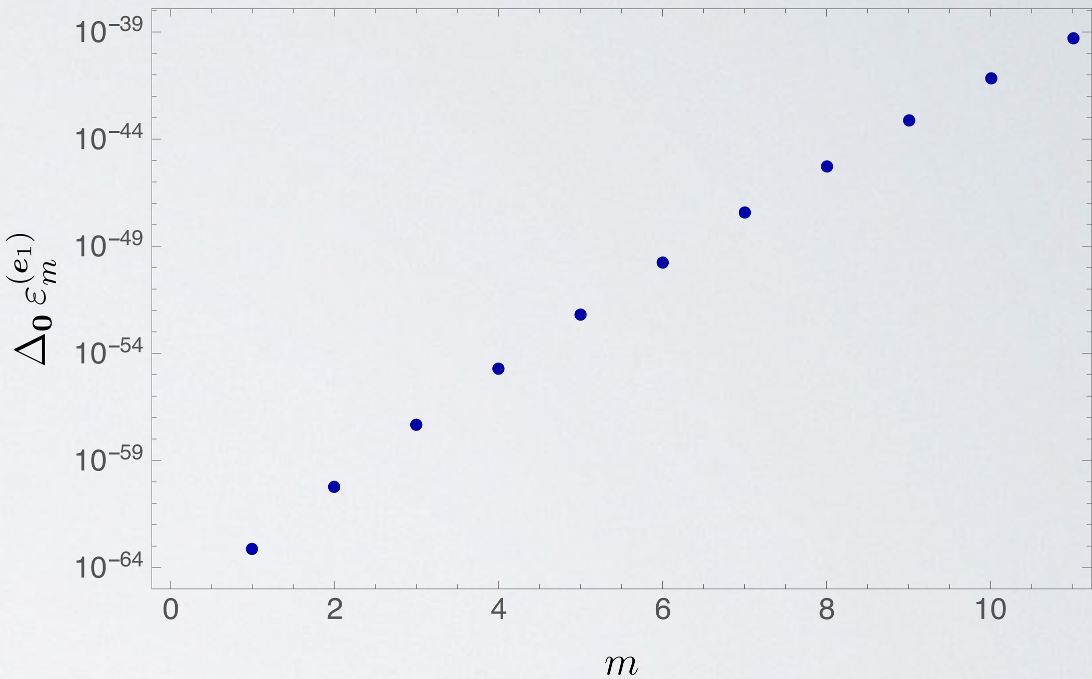
$$\omega_3 = \frac{3}{2}(6.769565 + 7.187931\mathrm{i})$$



ASYMPTOTIC ENERGY DENSITY

$$\mathcal{E}\left(u \equiv \tau^{2/3}, \boldsymbol{\sigma}\right) = \sum_{\boldsymbol{n}\in\mathbb{N}_0^{\infty}} \boldsymbol{\sigma}^{\boldsymbol{n}}\, \mathrm{e}^{-\boldsymbol{n}\cdot\boldsymbol{A}\,u}\, \Phi_{\boldsymbol{n}}(u)\ , \quad \Phi_{\boldsymbol{n}}(u) = u^{-\beta_{\boldsymbol{n}}} \sum_{k=0}^{+\infty} \varepsilon_k^{(\boldsymbol{n})}\, u^{-k}$$

- NP description of the late time behaviour of the SYM plasma
- Asymptotic analysis predicted coupled QMN solutions in gravity
- Agreement between gravity calculations and resurgence large-order predictions



[IA *et al*'18]

- Can we recover the non-equilibrium behaviour of early times?
- Dependence of the transseries parameters on initial conditions?

Study a simpler relativistic hydrodynamic system

3.

A SIMPLER EXAMPLE: MÜLLER-ISRAEL-STUART HYDRODYNAMICS

MIS CAUSAL HYDRODYNAMICS

Solve evolution equations of the Energy momentum tensor

$$\nabla_\mu T^{\mu\nu} = 0$$

- Assume ***boost invariant flow, conformal invariance***
- ***Hydrodynamic gradient expansion***: approximate shear stress tensor by corrections to ideal fluid

Müller-Israel-Stuart (MIS) equations

$$z\, C_{\tau\Pi} f\, f' + 4C_{\tau\Pi} f^2 + \left(z - \frac{16C_{\tau\Pi}}{3}\right) f - \frac{4C_\eta}{9} + \frac{16C_{\tau\Pi}}{9} - \frac{2z}{3} = 0$$

- Non-linear ODE describing the energy density
- $C_{\tau\Pi},\, C_\eta$ are phenomenological parameters

MIS CAUSAL HYDRODYNAMICS

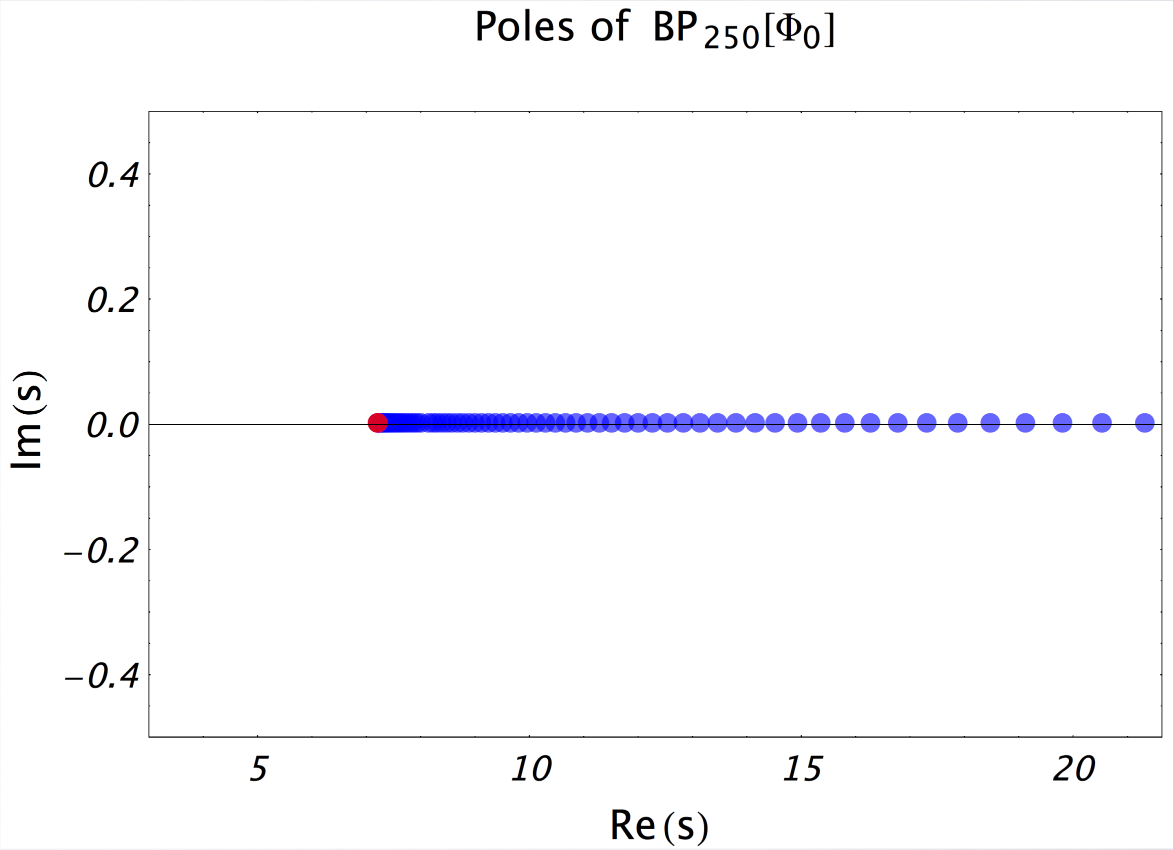
- We are interested in the late time regime $z \gg 1$
- It has a single, purely decaying non-hydrodynamic mode

Write the general solution as a transseries, sectors asymptotic.
Study resurgent properties

$$\mathcal{F}(z,\sigma) = \sum_{n=0}^{+\infty} \sigma^n \, \mathrm{e}^{-nAz} \, \Phi_n(z)$$

$$\Phi_n(z) = z^{-n\beta} \sum_{k=0}^{+\infty} a_k^{(n)} z^{-k}$$

$$A = \frac{3}{2C_{\tau\Pi}} \qquad \beta = -\frac{C_\eta}{C_{\tau\Pi}}$$



[Heller,Spalinski'15; Basar,Dunne'15; IA,Spalinski'15]

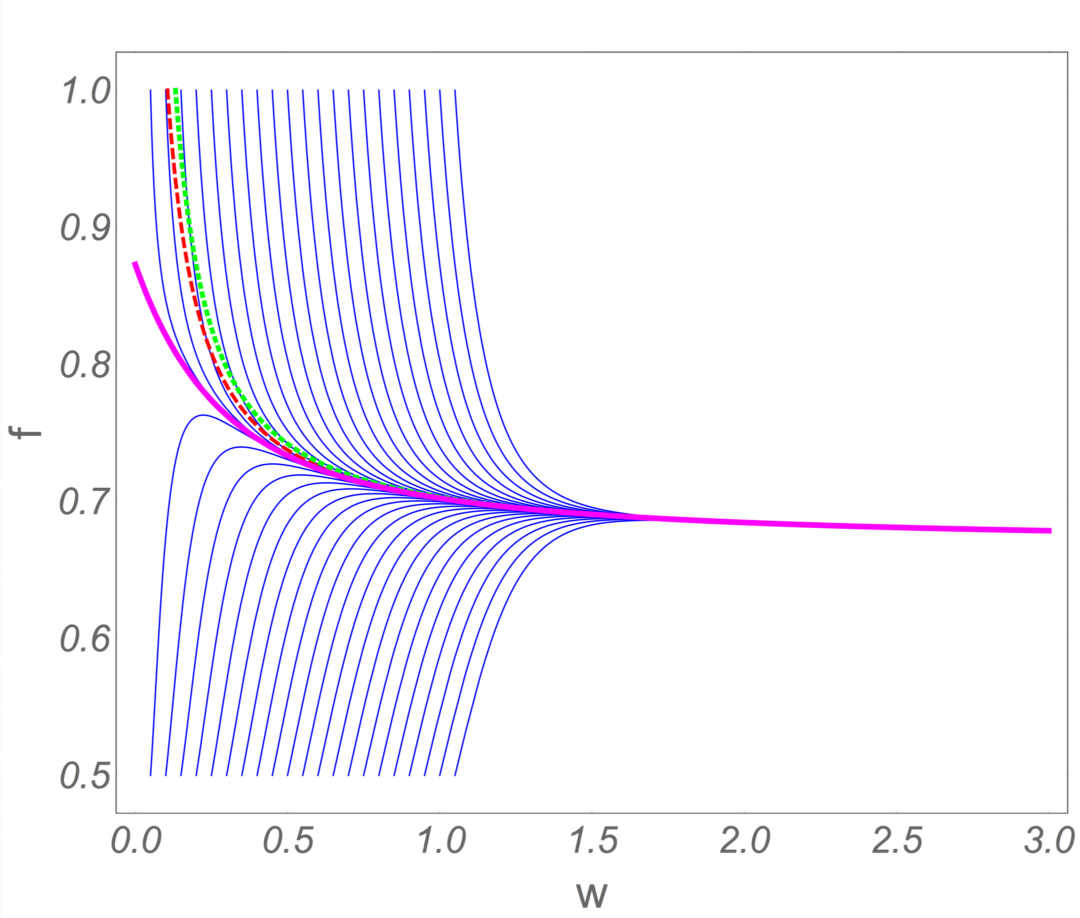
SOLUTION AT EARLY TIMES

Attractor solution: Stable solution, converging to a finite value at early times

Generic solution: divergent at early times, but will decay rapidly towards the attractor solution

$$\mathcal{F}_{\mathrm{Att}}(z) = \frac{2}{3} + \frac{1}{3}\sqrt{\frac{C_\eta}{C_{\tau\Pi}}} + \mathcal{O}(z)$$

Calculate attractor solution:
Taylor expansion



[Heller,Spalinski '15]

SOLUTION AT EARLY TIMES

$$\mathcal{F}(z,\sigma) = \sum_{n=0}^{+\infty} \sigma^n \, \mathrm{e}^{-nAz} \, \Phi_n(z) \qquad \Phi_n(z) = z^{-n\beta} \sum_{k=0}^{+\infty} a_k^{(n)} z^{-k}$$

Can we recover the attractor solution
from the transseries expansion?

Need to fix the value of $\sigma = \sigma_R + \mathrm{i}\sigma_I$

- Ambiguity cancelation fixes its imaginary part
- Comparison with attractor fixes its real part

[Heller,Spalinski '15]

AMBIGUITY CANCELATION

$$\mathcal{F}(z,\sigma) = \sum_{n=0}^{+\infty} \sigma^n \,\mathrm{e}^{-nAz}\, \Phi_n(z) \qquad\qquad \Phi_n(z) = z^{-n\beta} \sum_{k=0}^{+\infty} a_k^{(n)} z^{-k}$$

- positive real axis is a Stokes line
- Fix $\sigma = \sigma_R + \mathrm{i}\sigma_I$ such that summation below axis is

$$\mathcal{S}_-\mathcal{F}\left(z, \sigma_R + \frac{S}{2}\right)$$ with Stokes const: $S = -0.036537\,\mathrm{i}$

Ambiguity cancelation $\Rightarrow \quad \sigma_I = \dfrac{S}{2}$

ANALYTIC TRANSSERIES SUM

The order of transmonomials in the transseries can be rearranged:

$$\mathcal{F}(z,\sigma) = \sum_{k=0}^{+\infty} z^{-k} \sum_{n=0}^{+\infty} \left(\sigma\, z^{-\beta} \mathrm{e}^{-Az}\right)^n a_k^n$$

Define a new variable: $\tau = \sigma\, z^{-\beta} \mathrm{e}^{-Az}$

We want to sum the transseries in a new regime: $z^{-1} \ll \tau \ll 1$

The sum over powers of τ can be done exactly!

$$\mathcal{F}(z,\tau) = \sum_{k=0}^{+\infty} z^{-k} F_k(\tau) \qquad F_k(\tau) = \sum_{n=0}^{+\infty} \tau^n a_k^n$$

[Costin et al'01-13; IA,Schiappa,Vonk 'to appear]

ANALYTIC TRANSSERIES SUM

$$\mathcal{F}(z,\tau) = \sum_{k=0}^{+\infty} z^{-k}\, F_k(\tau) \qquad F_k(\tau) = \sum_{n=0}^{+\infty} \tau^n\, a_k^n$$

Recursive calculation:

$$F_0(\tau) = \frac{2}{3}\left(1 + W\left(\frac{3}{2}\tau\right)\right)$$

Lambert-W function

$$W(x)\,\mathrm{e}^{W(x)} = x$$

$$F_1(\tau) = \frac{1}{F_0(\tau)} \sum_{r=0}^{3} f_1^{(r)}(C_\eta, C_{\tau\Pi}) F_0(\tau)^r$$

$$\vdots$$

$$F_k(\tau) = \frac{P_k\left(F_0(\tau)\right)}{Q_k\left(F_0(\tau)\right)}$$

Polynomials

CONNECTION TO ATTRACTOR

$$\mathcal{F}(z,\tau) = \sum_{k=0}^{+\infty} z^{-k}\, F_k(\tau) \qquad\qquad z^{-1} \ll \tau \ll 1$$

Choose z large enough to be in above regime, but small enough to compare to attractor solution $\mathcal{F}_{\text{Att}}(z)$ at early times

- Choose z off the real axis $\quad z = z_R + \mathrm{i} z_I$
- Analytically continue attractor solution to complex plane

Solve $\quad \mathcal{F}(z,\tau) = \mathcal{F}_{\text{Att}}(z) \quad$ to obtain $\quad \tau(z) = \sum_{r\geq 0} \tau_r\, z^{-r}$

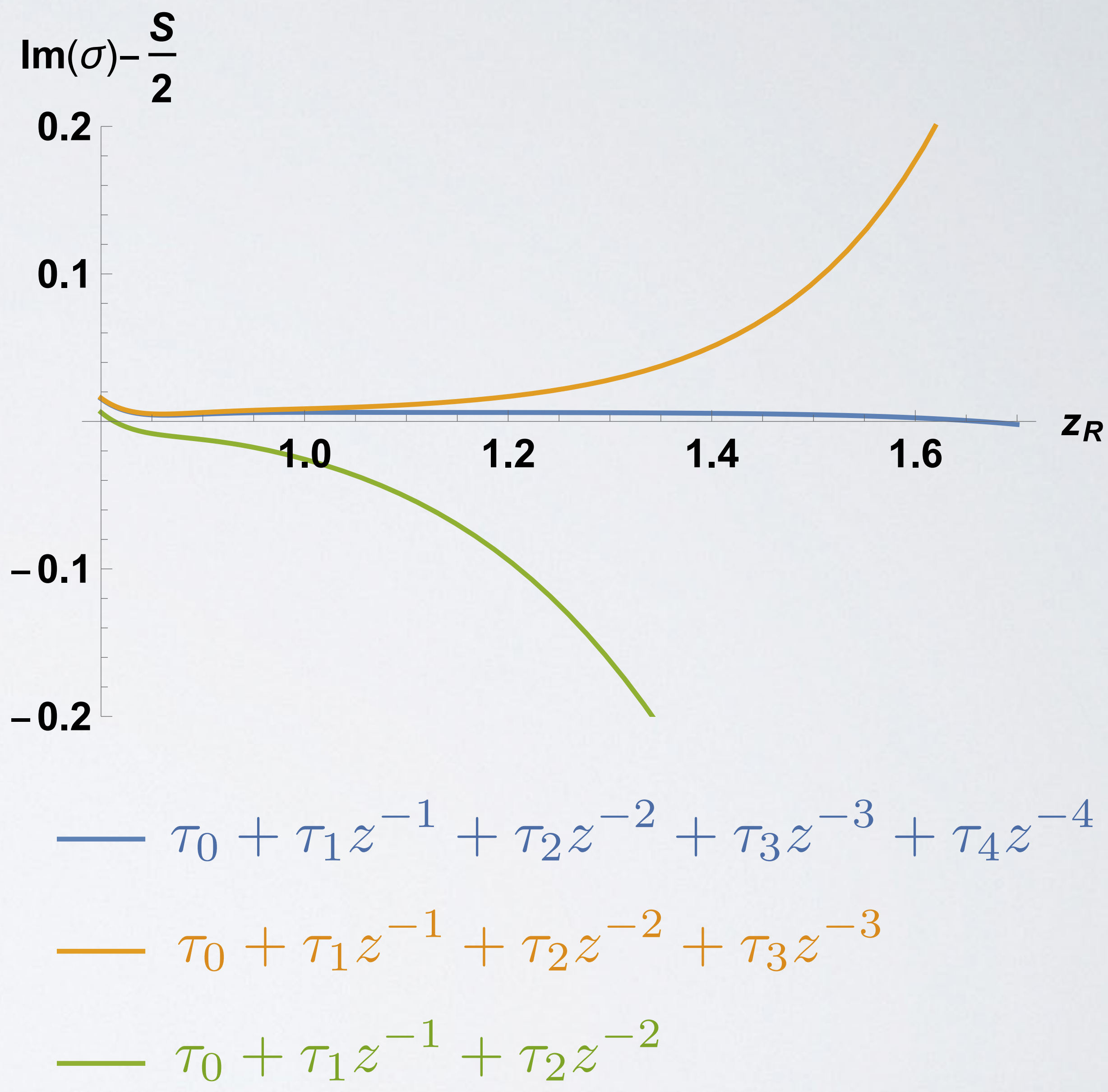
Transseries parameter: $\quad \sigma = z^{\beta} \mathrm{e}^{Az} \sum \tau_r\, z^{-r}$

CONNECTION TO ATTRACTOR

We obtain:

$$\sigma \sim -0.245 - 0.0128\mathrm{i}$$

Imaginary part approximates the value from ambiguity cancelation



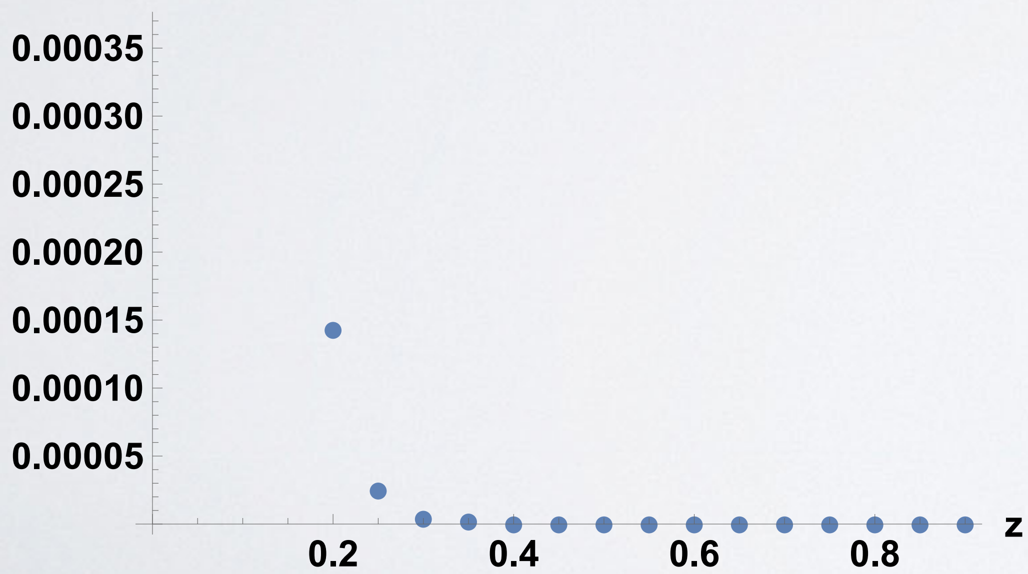
EARLY AND LATE TIMES

We can use the value obtained and perform Borel resummation:

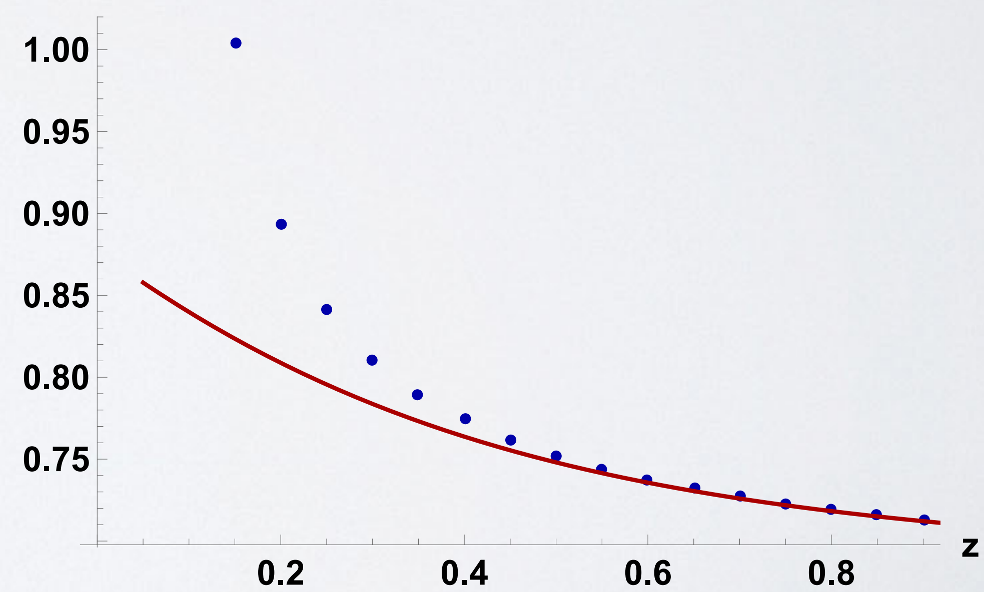
$$\mathcal{S}\mathcal{F}(z,\sigma) = \mathcal{S}_{-}\Phi_0(z) + \sigma\, \mathrm{e}^{-Az}\mathcal{S}_{-}\Phi_1(z) + \sigma^2\, \mathrm{e}^{-2Az}\mathcal{S}_{-}\Phi_2(z) + \cdots$$

with $\sigma \sim -0.245 - 0.0128\mathrm{i}$

$\mathcal{S}_{\mathrm{Im}}\mathcal{F}$



$\mathcal{S}_{\mathrm{Re}}\mathcal{F}$



4.
FURTHER APPLICATIONS

2D GRAVITY AND PAINLEVÉ I

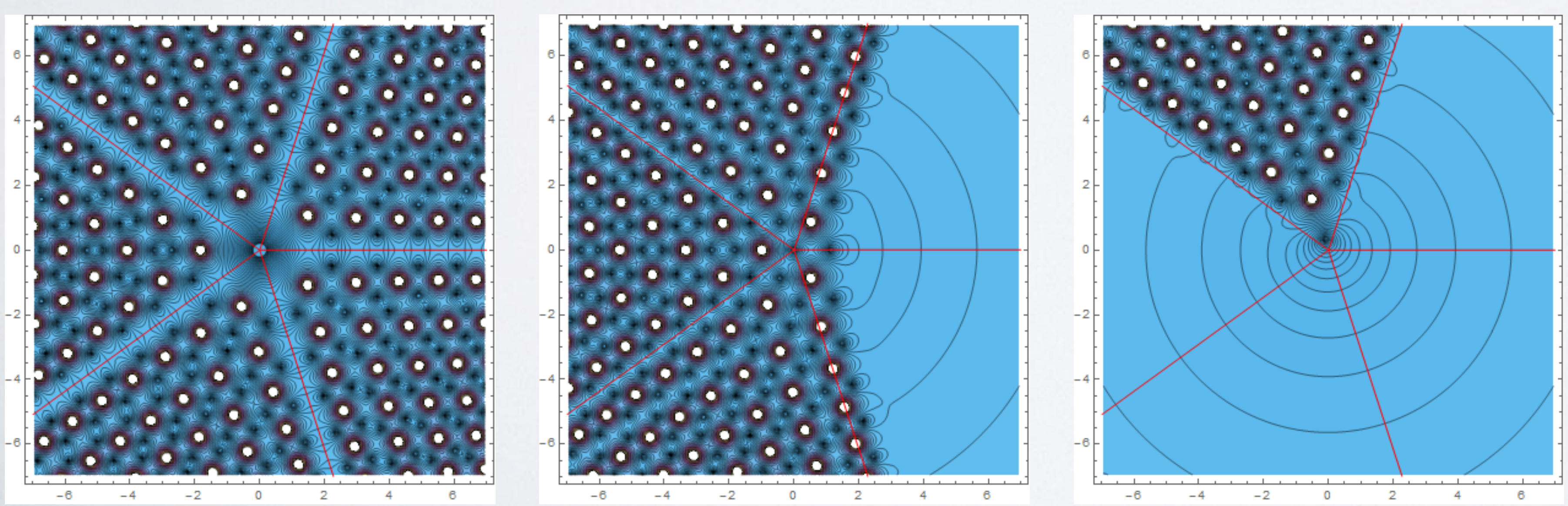
- Free energy of 2d gravity obeys Painlevé I ODE

$$F''(z) = -u(z) \qquad\qquad u^2(z) - \frac{1}{6}u''(z) = z$$

- Phase transitions: study Lee-Yang zeros of partition function

$$Z(z) = \mathrm{e}^{F(z)}$$

- Method: analytic prediction of poles of Painlevé transcendents from analytic transseries summation and Stokes phenomena



Structure of poles of tritronquée, tronquée and general solutions

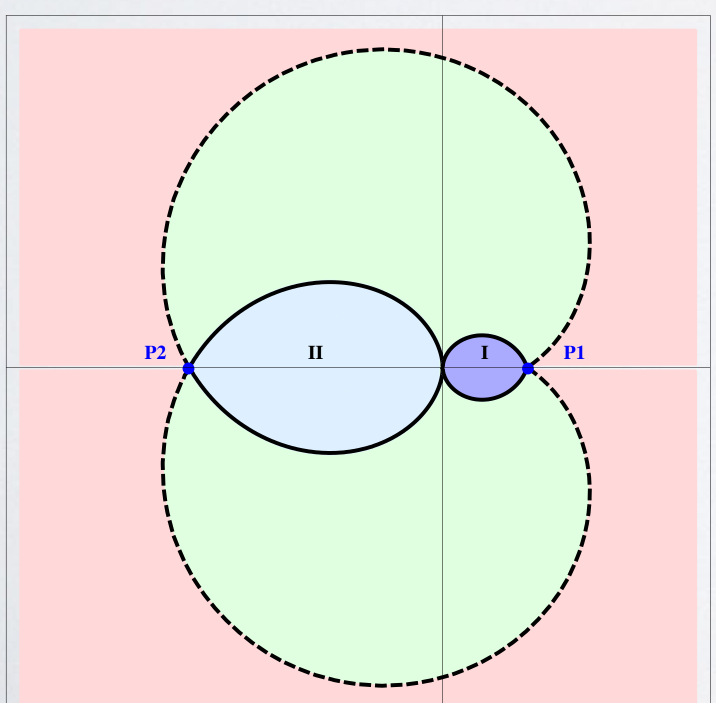
[IA,Schiappa,Vonk]

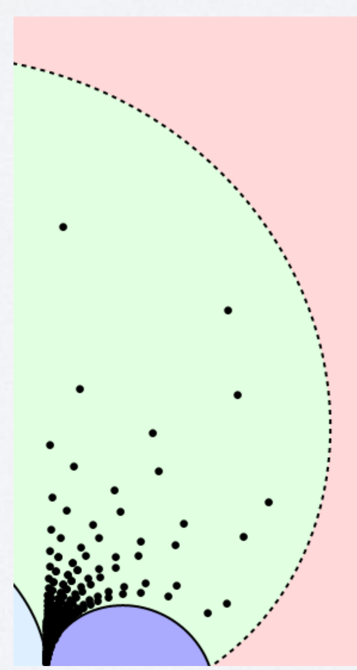
PHASE DIAGRAM: MATRIX MODELS

- Study the partition function of quartic matrix model

$$Z(N, g_s) \propto \int dM \, \exp\left(-\frac{1}{g_s}\mathrm{Tr}V(M)\right)\,, \quad V(z) = \frac{1}{2}z^2 - \frac{1}{24}\lambda z^4$$

- Large N expansion, phase diagram dependent on 't Hooft moduli
- Phase transitions: study Lee-Yang zeros of partition function
- Analytic transseries summation: determine position of zeros





phase diagram and position of zeros in anti-Stokes region

[IA,Schiappa,Vonk]

COLLECTION OF OPEN QUESTIONS

Transseries in gauge theories and matrix models

- asymptotics with multi-parameters
- interpretation of non-perturbative contributions

Determining Stokes constants

- play an essential role in ambiguity cancelation
- problem specific
- very few known analytically

Analytic properties of asymptotic observables

- phase transitions
- role of initial conditions
- boundary asymptotic matching

THANK YOU!

$$\sum_{n=0}^{\infty} E_n \, g^n \qquad e^{-A/g}$$