

Line defects in Omega-deformed M-theory

In collaboration with Jihwan Oh

Omega-deformed background

- Twisted supergravity background:

$$\mathbb{R} \times \mathbb{C}^2 \times \mathbb{R}_{\epsilon_1}^2 \times \mathbb{R}_{\epsilon_2}^2 \times \mathbb{R}_{\epsilon_3 \equiv -\epsilon_1 - \epsilon_2}^2$$

- Holomorphic-topological 5d theory.
- Perturbative description for $\epsilon_1 \ll \epsilon_2$:
5d non-commutative U(1) Chern-Simons gauge theory
$$\frac{1}{\epsilon_1} \int [AdA + A(A *_{\epsilon_2} A)] dz_1 dz_2$$

Omega deformed defects

- Topological line defects: M2 branes on $\mathbb{R} \times \mathbb{R}_{\epsilon_i}^2$
- Holomorphic surface defects: M5 branes on $\mathbb{C} \times \mathbb{R}_{\epsilon_i}^2 \times \mathbb{R}_{\epsilon_j}^2$
- Intersections of M2's and M5's

Objectives

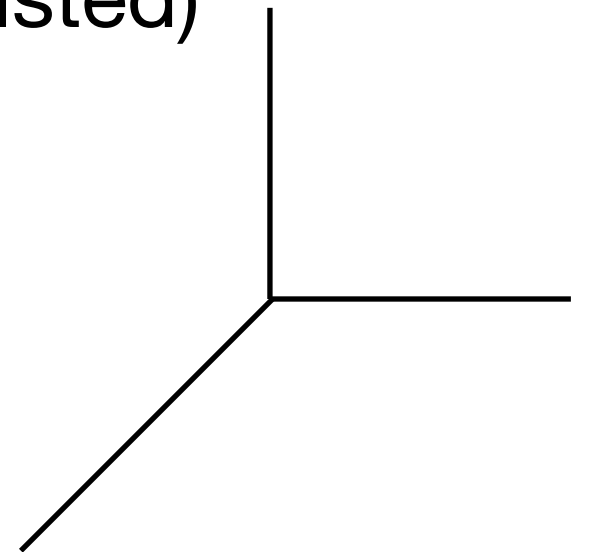
- Understand math tools: Koszul duality, relation to holography.
- Understand triality.
- Study M2-M5 intersections and relations to corner VOA
- Preparation for general toric CY

Omega deformation and string duality

- Costello: reduce to IIA on a circle in $\mathbb{R}_{\epsilon_2}^2 \times \mathbb{R}_{\epsilon_3}^2$
- Omega-deformed D6 gives 5d CS theory
- M2 on $\mathbb{R} \times \mathbb{R}_{\epsilon_1}^2$ give D2 and then Nf=1 ADHM
 - Other M2's give Wilson lines (not in Costello)
- M5 on $\mathbb{C} \times \mathbb{R}_{\epsilon_2}^2 \times \mathbb{R}_{\epsilon_3}^2$ give D4 and then chiral fermions
 - Other M5's give 't Hooft surfaces (disorder operators)

Omega deformation and string duality continued

- Alternative: reduce on two-torus to IIB (GL twisted)
 - Fivebrane web from Toric diagram
 - 5d CS from junction
 - M5's to D3's and then corner VOA's
 - M2's to strings/D1/(1,1) branes



Omega deformation and chiral algebras

- Original application: 4d N=2 on $\mathbb{R}_{\epsilon_1}^2 \times \mathbb{R}_{\epsilon_2}^2$
 - Localizes to 0d integral on instanton moduli space
$$Z_{\epsilon_1, \epsilon_2} = e^{\frac{1}{\epsilon_1 \epsilon_2} \mathcal{F} + \dots}$$
 - AGT: in class S, partition function is a conformal block
- 6d explanation: 6d SCFT on $\mathbb{C} \times \mathbb{R}_{\epsilon_i}^2 \times \mathbb{R}_{\epsilon_j}^2$ reduces to $\mathcal{W}_{\epsilon_i, \epsilon_j}^{\mathfrak{g}}$ chiral algebra
 - M5 surface defect requires coupling to $\mathcal{W}_{\epsilon_i, \epsilon_j}^{u(N)}$

Omega deformation and quantized Higgs branch

- Second traditional application: (2,2) on \mathbb{R}_ϵ^2
 - Localize to 0d. Partition function related to superpotential
- 2d analogue of AGT: J-functions from finite W algebras
 - 3d explanation: 3d N=4 on $\mathbb{R} \times \mathbb{R}_\epsilon^2$
 - Topological 1d theory, “quantization” of Higgs branch
- Application to M2 worldvolume: Nf=1 ADHM quiver

Surface defect coupling to 5d CS theory

- Surface defect at $t=z=0$:

$$\sum_{n \geq 0} \int_{\mathbb{C}_w} W^{(n)}(w) \partial_z^n A_{\bar{w}} \big|_{t=z=0}$$

- Infinite tower of currents, OPE fixed by non-commutative $U(1)$ gauge algebra
- Classical $\mathcal{W}_{1+\infty}$

Surface defect coupling to 5d CS theory, continued

- Conjecture: non-zero ϵ_1 deforms OPE to quantum $\mathcal{W}_{1+\infty}^{\epsilon_1, \epsilon_2}$
- Test: quantum $\mathcal{W}_{1+\infty}$ has hidden triality symmetry!
- Test: for central element $\lambda = \epsilon_3 N$ truncation to $\mathcal{W}_{\epsilon_1, \epsilon_2}^{u(N)}$
- Test: corner VOA $Y_{\epsilon_1, \epsilon_2}^{n_1, n_2, n_3}$ from 3 stacks of M5 for $\lambda = \sum_i n_i \epsilon_i$

Line defect coupling to 5d CS theory

- Line defect at $z=w=0$:

$$\sum_{n,m} \int_{\mathbb{R}_t} T_{n,m}(t) \partial_z^n \partial_w^m A_t|_{z=w=0}$$

- Infinite tower of worldliness operators, algebra fixed by non-commutative $U(1)$ gauge algebra
- Classical $U(\text{Diff}_{\mathbb{C}}^{\epsilon_2})$

Line defect coupling to 5d CS theory, continued

- Previously known: quantum effects deform $U(\text{Diff}_{\mathbb{C}}^{\epsilon_2})$ to some specific $\mathcal{A}_{\epsilon_1, \epsilon_2}$
 - Determined as uniform-in-N limit of Nf=1 ADHM quiver
Quantum Higgs branches $\mathcal{A}_{\epsilon_1, \epsilon_2}^{(N)}$
- Our result: $\mathcal{A}_{\epsilon_1, \epsilon_2}$ is triality invariant!
 - Compatible with truncations for 3 types of M2 branes

More on triality invariance

- Hard to show in ADHM Higgs branch
- Trick: mirror symmetry to ADHM Coulomb branch
 - Explicit triality-invariant presentation
- Triality invariant connection: $A = \epsilon_2 \Phi$
$$S[\Phi] \sim \frac{1}{\epsilon_1 \epsilon_2 \epsilon_3} \int \Phi d\Phi + \Phi \{\Phi, \Phi\} + \dots$$
 - “Symplectic gravity”?

Modules and M2-M5 junctions

- Conjecture: endpoints of M2's on M5's are Verma modules for $\mathcal{A}_{\epsilon_1, \epsilon_2}^{(N)}$
 - Labelled by Young diagrams
 - Coulomb branch vortex construction
 - Uniform-in-N description is possible
 - Same labels as M2 modules in corner VOA!
- We also have conjecture for M2 passing through M5's

Open problems

- Extract physics from Omega-deformed theory
- Which BPS configurations localize?
- Explicit worldline theory for multiple M2 stacks
- Full comparison between Verma modules and M2 chiral modules
- General M2-M5 intersections
- R-matrices, coproducts, fusion of defects, etc.
- General toric CY, gluing 5d theories along webs.