Line defects in Omega-deformed M-theory

In collaboration with Jihwan Oh
Omega-deformed background

- Twisted supergravity background:
  \[ \mathbb{R} \times \mathbb{C}^2 \times \mathbb{R}^2_{\epsilon_1} \times \mathbb{R}^2_{\epsilon_2} \times \mathbb{R}^2_{\epsilon_3=\epsilon_1-\epsilon_2} \]

- Holomorphic-topological 5d theory.

- Perturbative description for \( \epsilon_1 \ll \epsilon_2 \):
  5d non-commutative U(1) Chern-Simons gauge theory
  \[ \frac{1}{\epsilon_1} \int \left[ AdA + A(A \ast_{\epsilon_2} A) \right] dz_1 dz_2 \]
Omega deformed defects

- Topological line defects: M2 branes on $\mathbb{R} \times \mathbb{R}^2_{\epsilon_i}$

- Holomorphic surface defects: M5 branes on $\mathbb{C} \times \mathbb{R}^2_{\epsilon_i} \times \mathbb{R}^2_{\epsilon_j}$

- Intersections of M2’s and M5’s
Objectives

• Understand math tools: Koszul duality, relation to holography.

• Understand triality.

• Study M2-M5 intersections and relations to corner VOA

• Preparation for general toric CY
Omega deformation and string duality

- Costello: reduce to IIA on a circle in $R^2_{\epsilon_2} \times R^2_{\epsilon_3}$
- Omega-deformed D6 gives 5d CS theory
- $M2$ on $R \times R^2_{\epsilon_1}$ give D2 and then Nf=1 ADHM
  - Other M2’s give Wilson lines (not in Costello)
- $M5$ on $C \times R^2_{\epsilon_2} \times R^2_{\epsilon_3}$ give D4 and then chiral fermions
  - Other M5’s give ’t Hooft surfaces (disorder operators)
Omega deformation and string duality continued

- Alternative: reduce on two-torus to IIB (GL twisted)
  - Fivebrane web from Toric diagram
  - 5d CS from junction
  - M5’s to D3’s and then corner VOA’s
  - M2’s to strings/D1/(1,1) branes
Omega deformation and chiral algebras

- Original application: 4d N=2 on $\mathbb{R}^2_{\epsilon_1} \times \mathbb{R}^2_{\epsilon_2}$

- Localizes to 0d integral on instanton moduli space
  $$Z_{\epsilon_1, \epsilon_2} = e^{\frac{1}{\epsilon_1 \epsilon_2} \mathcal{F} + \cdots}$$

- AGT: in class S, partition function is a conformal block

- 6d explanation: 6d SCFT on $\mathbb{C} \times \mathbb{R}^2_{\epsilon_i} \times \mathbb{R}^2_{\epsilon_j}$ reduces to $\mathcal{W}^g_{\epsilon_i, \epsilon_j}$ chiral algebra

- M5 surface defect requires coupling to $\mathcal{W}^u(N)_{\epsilon_i, \epsilon_j}$
Omega deformation and quantized Higgs branch

- Second traditional application: (2,2) on $\mathbb{R}_\epsilon^2$
  - Localize to 0d. Partition function related to superpotential

- 2d analogue of AGT: J-functions from finite W algebras
  - 3d explanation: 3d N=4 on $\mathbb{R} \times \mathbb{R}_\epsilon^2$

- Topological 1d theory, "quantization" of Higgs branch

- Application to M2 worldvolume: Nf=1 ADHM quiver
Surface defect coupling to 5d CS theory

- Surface defect at $t=z=0$:
  \[ \sum_{n \geq 0} \int_{\mathbb{C}_w} W^{(n)}(w) \partial_z^n A_{\bar{w}} \big|_{t=z=0} \]

- Infinite tower of currents, OPE fixed by non-commutative U(1) gauge algebra

- Classical $\mathcal{W}_{1+\infty}$
Surface defect coupling to 5d CS theory, continued

• Conjecture: non-zero $\epsilon_1$ deforms OPE to quantum $\mathcal{W}^{\epsilon_1,\epsilon_2}_{1+\infty}$

• Test: quantum $\mathcal{W}_{1+\infty}$ has hidden triality symmetry!

• Test: for central element $\lambda = \epsilon_3 N$ truncation to $\mathcal{W}^{u(N)}_{\epsilon_1,\epsilon_2}$

• Test: corner VOA $\mathcal{Y}^{n_1,n_2,n_3}_{\epsilon_1,\epsilon_2}$ from 3 stacks of M5 for $\lambda = \sum_i n_i \epsilon_i$
Line defect coupling to 5d CS theory

- Line defect at $z=w=0$:
  \[ \sum_{n,m} \int_{\mathbb{R}^t} T_{n,m}(t) \partial_z^n \partial_w^m A_t \bigg|_{z=w=0} \]

- Infinite tower of worldliness operators, algebra fixed by non-commutative $U(1)$ gauge algebra

- Classical $U(\text{Diff}_C^{\epsilon_2})$
Line defect coupling to 5d CS theory, continued

- Previously known: quantum effects deform $U(\text{Diff}_C^{\epsilon_2})$ to some specific $A_{\epsilon_1,\epsilon_2}$
- Determined as uniform-in-N limit of Nf=1 ADHM quiver Quantum Higgs branches $A_{(N)}^{\epsilon_1,\epsilon_2}$
- Our result: $A_{\epsilon_1,\epsilon_2}$ is triality invariant!
  - Compatible with truncations for 3 types of M2 branes
More on triality invariance

- Hard to show in ADHM Higgs branch
- Trick: mirror symmetry to ADHM Coulomb branch
  - Explicit triality-invariant presentation
- Triality invariant connection: $A = \epsilon_2 \Phi$

$$S[\Phi] \sim \frac{1}{\epsilon_1 \epsilon_2 \epsilon_3} \int \Phi d\Phi + \Phi\{\Phi, \Phi\} + \cdots$$

- "Symplectic gravity"?
Modules and M2-M5 junctions

- Conjecture: endpoints of M2’s on M5’s are Verma modules for $A^{(N)}_{\epsilon_1, \epsilon_2}$

- Labelled by Young diagrams

- Coulomb branch vortex construction

- Uniform-in-N description is possible

- Same labels as M2 modules in corner VOA!

- We also have conjecture for M2 passing through M5’s
Open problems

- Extract physics from Omega-deformed theory
- Which BPS configurations localize?
- Explicit worldline theory for multiple M2 stacks
- Full comparison between Verma modules and M2 chiral modules
- General M2-M5 intersections
- R-matrices, coproducts, fusion of defects, etc.
- General toric CY, gluing 5d theories along webs.