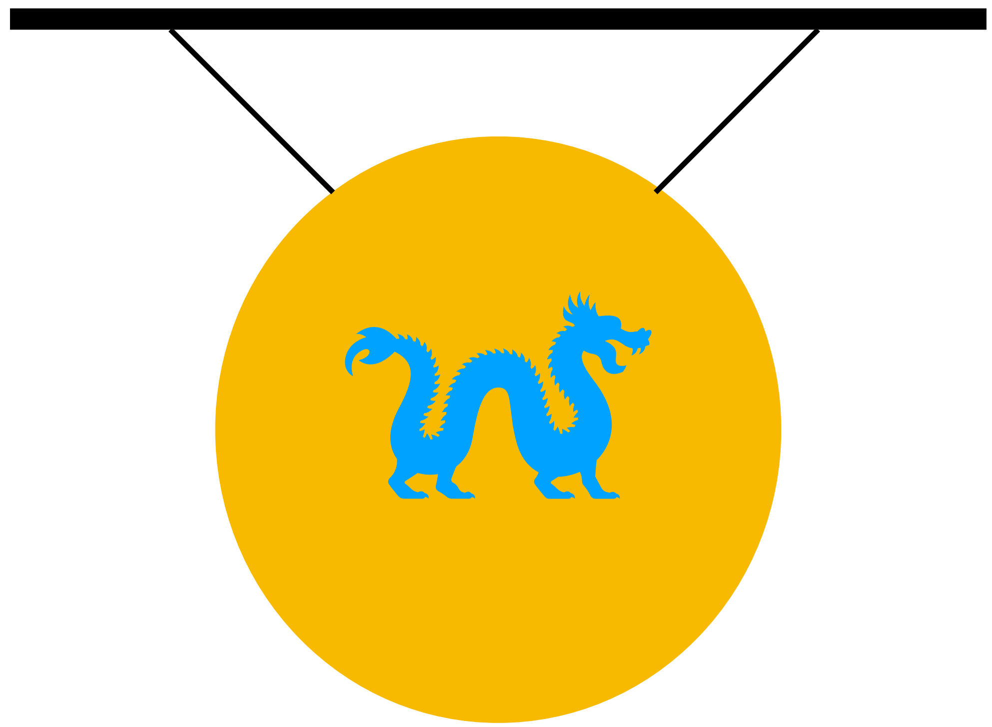
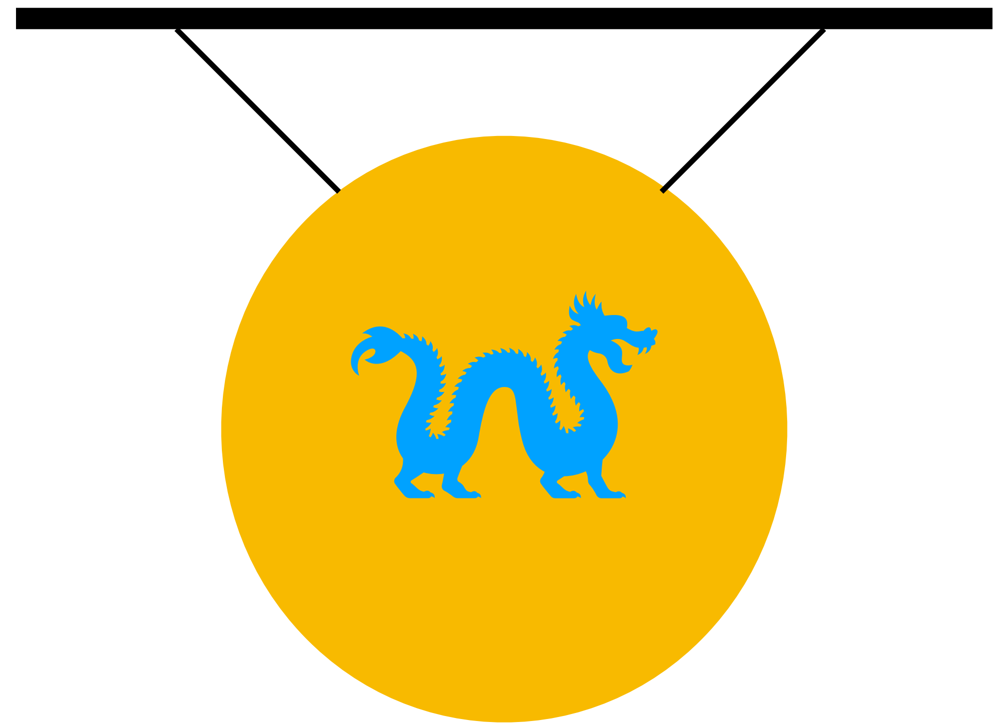
STRING MATH 2019

01-05 JULY 2019, UPPSALA, SWEDEN



Gong Show

With your host Kimyeong Lee

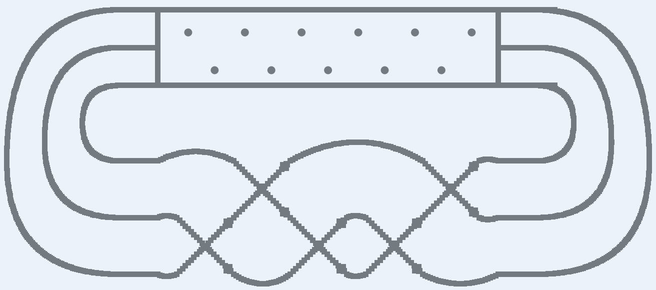
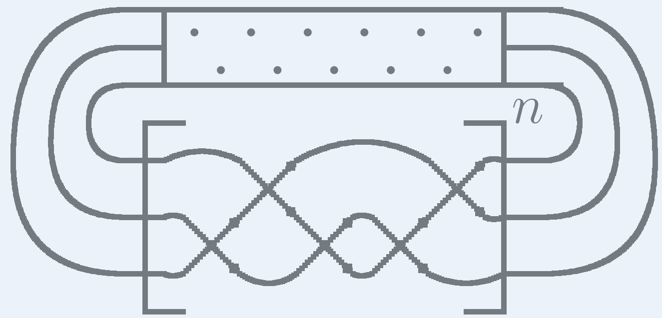


1

A. Anokhina, ITEP, Moscow
Joint project with P.Dunin-Barkowski, A.Morozov, and A.Popolitov

(Multi-)linear evolution for knot (homology) polynomials

$$P[\mathcal{K}_n] = \sum_{\Lambda} c_\Lambda \Lambda^n = \mathrm{Tr}\hat{U}^n$$

The knot	Evolution	Family of knots
$\mathcal{K}$		$[\mathcal{K}_n]$
	$\longrightarrow$	

$\hat{\mathcal{R}}$-matrices $\mapsto$ Jones/HOMFLY $\mapsto$ Linear evolution

$\updownarrow$ $\updownarrow$

??? $\mapsto$ Khovanov/-Rozanski $\mapsto$ Linear evolution (deformed)

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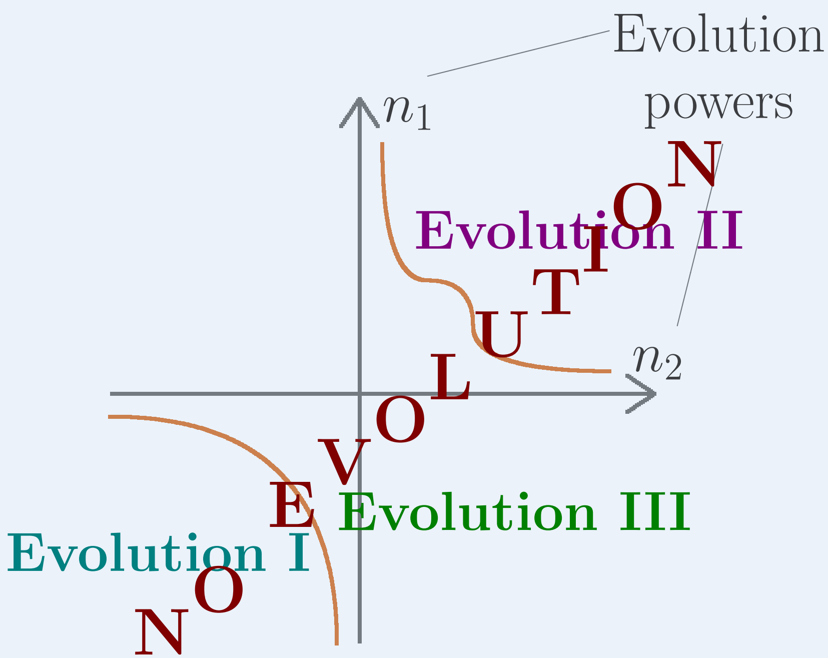
$$P[\mathcal{K}_{n_1,n_2}] = \sum_{\Lambda_1,\,\Lambda_2} c_{\Lambda_1,\Lambda_2}\,\Lambda_1^{n_1}\Lambda_2^{n_2}$$

No go theorems $\Rightarrow$ no global evolution for Khovanov(-Rozansky)

The answer: set of local evolutions on separate domains



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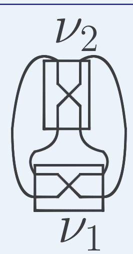
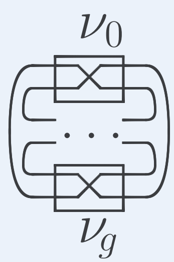
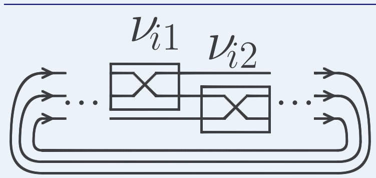
(Multi-)linear evolution for knot (homology) polynomials

$$P[\mathcal{K}_n] = \sum_{\Lambda} c_\Lambda \underset{\uparrow}{\Lambda^n}$$

Deformation of the eigenvalues

	**Plain**	$q^2 \to q^2 t$
	**Nimble**	$q^2 \to \{q^2 t, (q^2 t)^2\}$
	**Freaky**	$f(q) \to \{\tilde{f}_a(q,t)\}$

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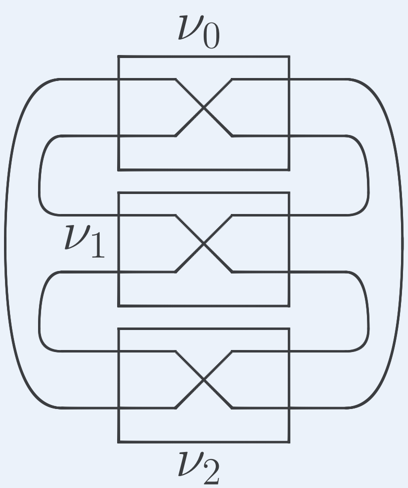
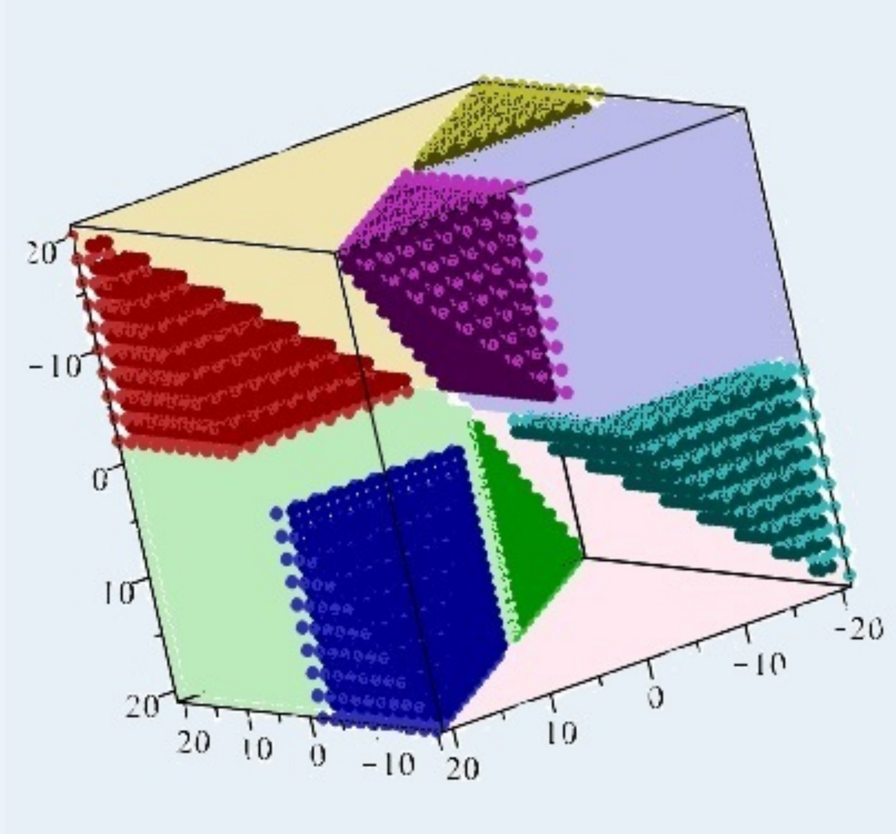
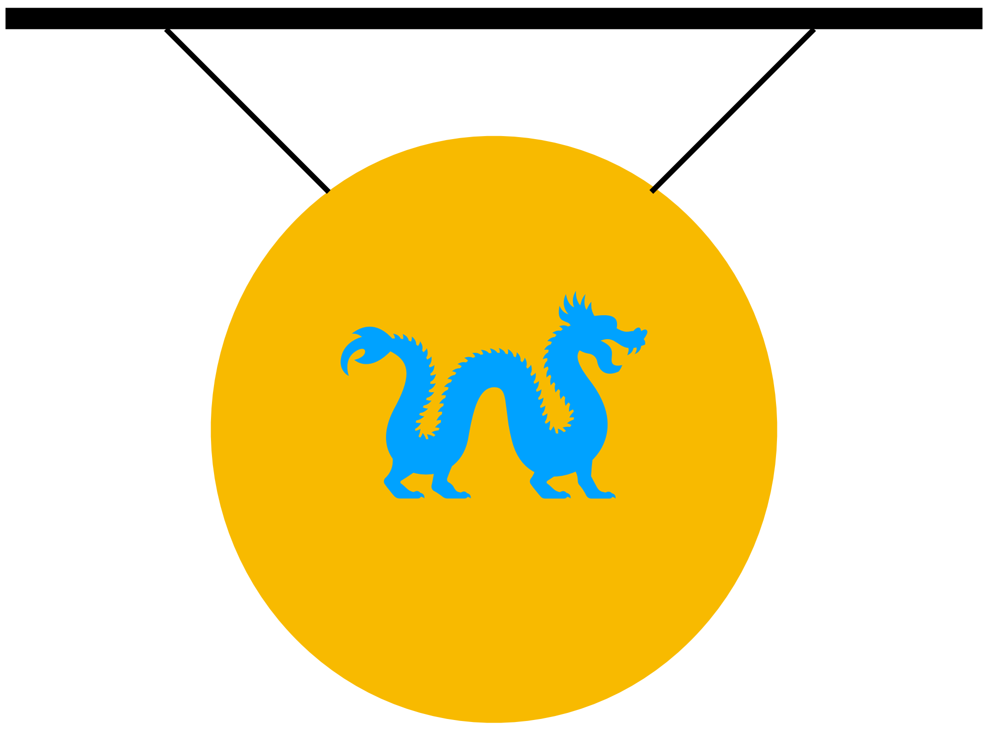
Example of 3d phase diagram. Genus 2 pretzel knots.





Same color=same evolution formula.

Thanks to Sh. Shakirov for improving this picture!



2

Index Expressions for Line Bundle Cohomology

Callum Brodie

University of Oxford

Based on 1906.08363, 1906.08730, and 1906.08769

with Andrei Constantin, Rehan Deen, and Andre Lukas

1st of July 2019

Motivation

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Vector bundle cohomology is crucial information in many string theory applications.

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One common and important class is line bundles,
e.g. in realistic model-building.

Currently computation is difficult, doesn't provide insight,
(algorithmic, computer-based, time consuming).

Deeper understanding would be very useful,
e.g. for classification, bottom-up model-building, ...

Hints from ‘experiment’

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Cohomologies described by piecewise polynomial formulae:

[Constantin, Lukas ‘18], [Larfors, Schneider ‘19], [CB, Constantin, Deen, Lukas ‘19]

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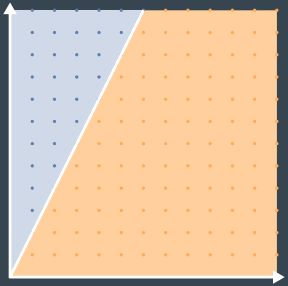
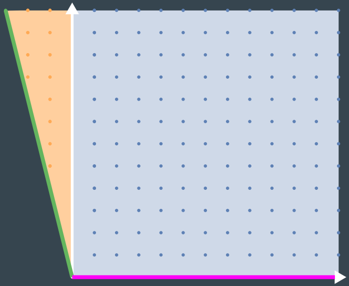
Two-folds
(del Pezzos, toric, ...)



Three-folds
(CICYs, toric hyp.)



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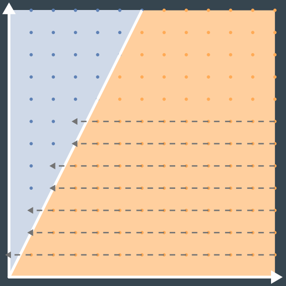

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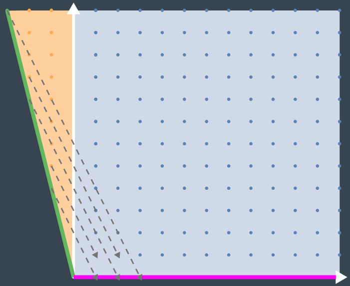

Key observation:

Polynomials described by map into a ‘fundamental’ region.

Results

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Maps for h^0 drop 'rigid' pieces of divisor associated to bundle, mapping to nef cone (Zariski decomposition).

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$\Rightarrow$ Index expressions for h^0 for e.g. all del Pezzos:

$$h^0\left(\mathrm{dP}_n, \mathcal{O}_{\mathrm{dP}_n}(D)\right) = \chi\left(D + \sum_{\{i|C_i^2=-1\}} \#_i \, C_i\right),$$

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Covers many three-fold cases:

Can lift surface cohomology to $h^{0,1,2,3}$ on elliptic CY3.

Some applications

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- Bottom-up model-building

  Some CY3 now already covered, e.g. elliptic

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- Bottom-up model-building

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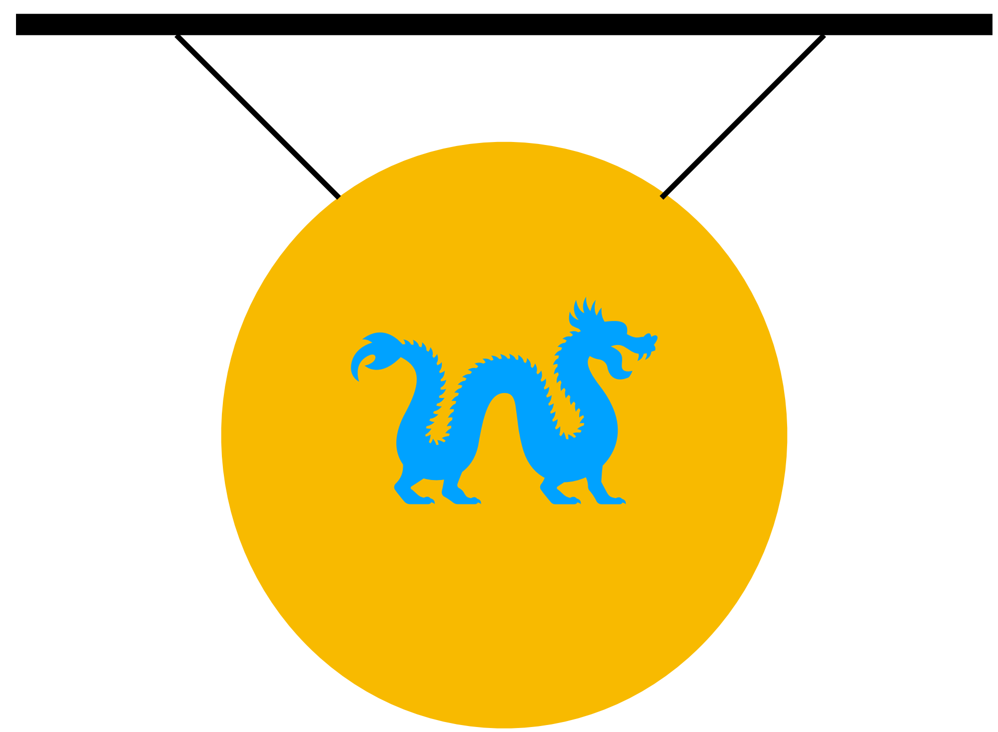
- Jumping loci

  Systematic understanding, all bundles at once

- Reverse-engineering rigid divisors

  Rigid divisors seen in region edges, find by e.g. ML



3

Higgs Bundles for M-theory on G_2-manifolds

Sebastjan Cizel
Joint with Andreas Braun, Max Hübner and Sakura Schäfer-Nameki

StringMath 2019, Uppsala
July 1, 2019

UNIVERSITY OF OXFORD

M-theory and 7d SYM

M-theory on G_2-manifolds gives a $\mathcal{N}=1$ theory in 4d (with matter, coupled to SUGRA)

For interesting 4d physics one needs G_2-spaces with singularities in codimension 4 and 7. Compact examples not known.

Instead consider a (noncompact) local limit [Pantev, Wijnholt]

$$\mathbb{C}^2/\Gamma_{\mathrm{ADE}} \hookrightarrow X_7 \to M_3\,, \qquad M_3 \text{ associative.}$$

M-theory reduced on the $\mathbb{C}^2/\Gamma_{\mathrm{ADE}}$ fibre

$\Downarrow$

partially twisted 7d SYM on $\mathbb{R}^{1,3} \times M_3$ with ADE gauge group

7d SYM contains a Higgs field $\phi \in \Omega^1(\mathrm{ad}(\mathfrak{g}_{\mathrm{ADE}}))$. BPS equations give the Hitchin system on M_3

$$F_W - i[\phi, \phi] = 0\,, \qquad D_W \phi = 0\,, \qquad D_W^\dagger \phi = 0\,.$$

Computing the Chiral Spectrum

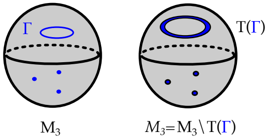
Simplified problem ($[\phi, \phi] = 0$, $\pi_1(M_3) = 0$) reduces to

$$\phi = df\,, \qquad \text{with} \quad \Delta f = 0 \quad \Rightarrow \text{no non-constant solutions.}$$

Introduce a source term ρ along $\Gamma \subset M_3$ and consider

$$\Delta f = \rho, \qquad \text{with} \qquad \int_{M_3} \rho = 0.$$

Excise tubular neighbourhood
of $\Gamma = \Gamma_+ \cup \Gamma_- \subset M_3$
(configuration of charges)
to get $\mathcal{M}_3$ with boundary $\Sigma_+ \cup \Sigma_-$.



Impose boundary conditions

$$\text{Dirichlet on } \Sigma_-, \qquad\qquad \text{Neumann on } \Sigma_+\,.$$

Chiral spectrum computed by the relative cohomology of a pair $(\mathcal{M}_3, \Sigma_-)$

$$\text{chiral} : H^1(\mathcal{M}_3, \Sigma_-), \quad \text{conj. chiral} : H^2(\mathcal{M}_3, \Sigma_-)\,.$$

Localised Matter

Matter is localised at $\phi = df = 0$ i.e. critical loci of f and the chiral fermions are in the kernel of

$$\Delta_f = \mathcal{D}_f \mathcal{D}_f^\dagger + \mathcal{D}_f^\dagger \mathcal{D}_f \,, \qquad \mathcal{D}_f = d + df \wedge$$

[Witten]: This is Hamiltonian for an SQM and it computes the Morse cohomology.

Motivated by TCS: What if f has 1d critical loci i.e. is Morse-Bott?

SQM model $\Rightarrow$ Morse-Bott cohomology $\Rightarrow$ recovers $H^*(\mathcal{M}_3, \Sigma_-)$

One gains more information from Morse(-Bott) picture:

- gradient curves lift to associatives in the total space of the ALE fibration $\Rightarrow$ M2-branes
- Yukawa interactions (and higher couplings) can be expressed in terms of gradient flow trees.

Chiral Spectrum of Twisted Connected Sum

Twisted connected sum G_2-manifolds are built from two building blocks

$$K3 \;\hookrightarrow\; X_{\pm} \;\to S^1 \times \mathbb{C}_{\pm}.$$

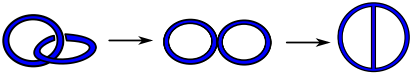
$\Rightarrow$ the Higgs field $\phi_{\pm} = df_{\pm}$ is S^1-invariant $\Rightarrow$ critical loci are (only) circles.

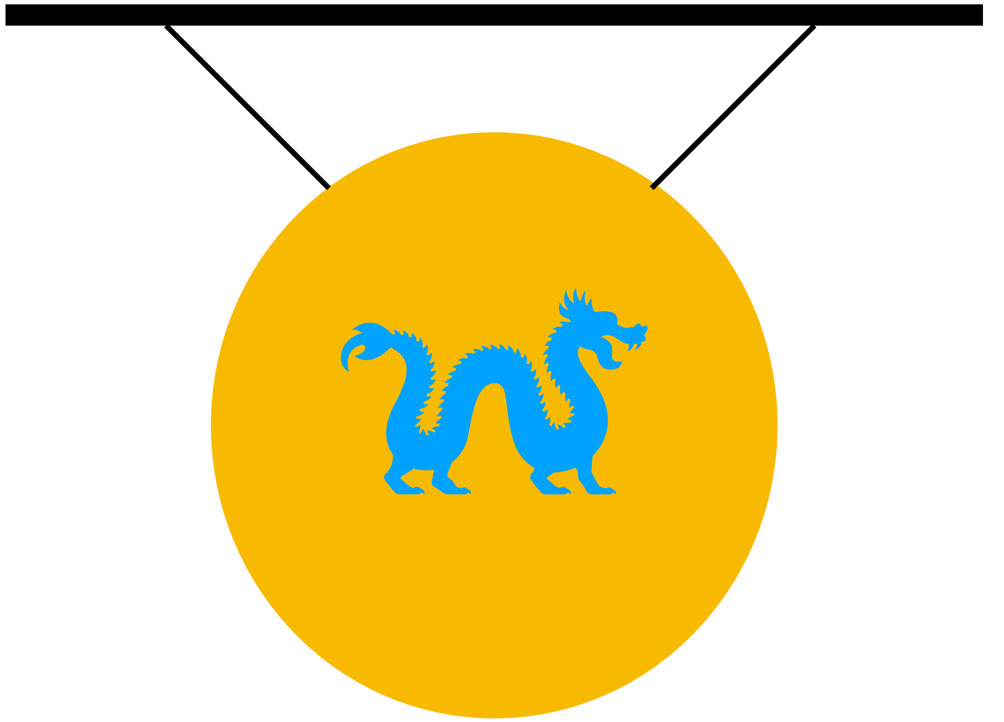
Using Morse-Bott theory we can show that in this case

$$\text{chiral index} \equiv 0\,.$$

Hence, TCS compactifications do not give rise to chiral spectra.

However, in the local model, one can deform charge configuration to give chiral spectrum.





4

Topological Matter and Langlands Program

Kazuki Ikeda
Osaka U.

String Math 2019

K.I, arXiv 1812.11879 (2018)

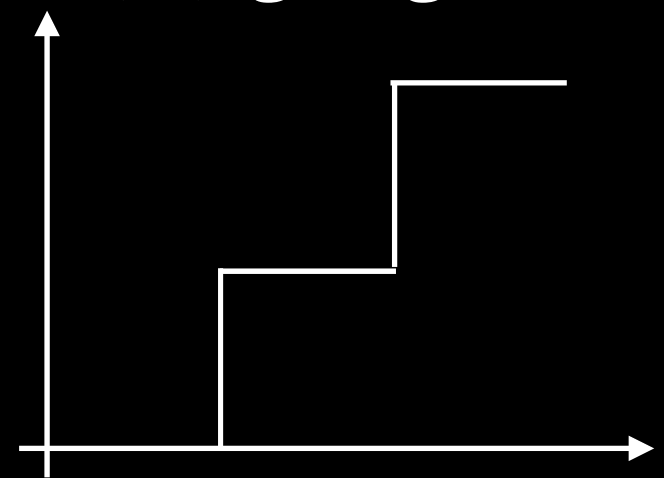
K.I, Annals of Physics, 397, 136 (2018)

K.I, Journal of Mathematical Physics, 59, 061704 (2018)

Quantum Hall Effect: U(1) gauge theory on a torus

conductance

σ_{xy}



magnetic flux ϕ

$$\sigma_{xy} = \frac{e^2}{h}\nu$$

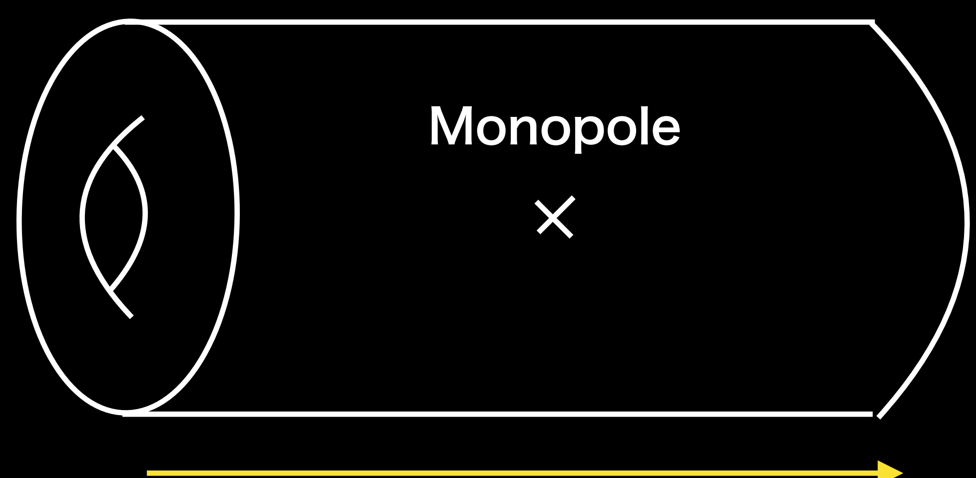
$$\nu = c_1(\mathcal{L})$$

Geometric Langlands

Line bundles on T^2

$$\mathcal{L} = (\mathcal{L}, \nabla) \longrightarrow \mathcal{F}_{\mathcal{L}}$$

Hecke eigensheaves



magnetic flux ϕ

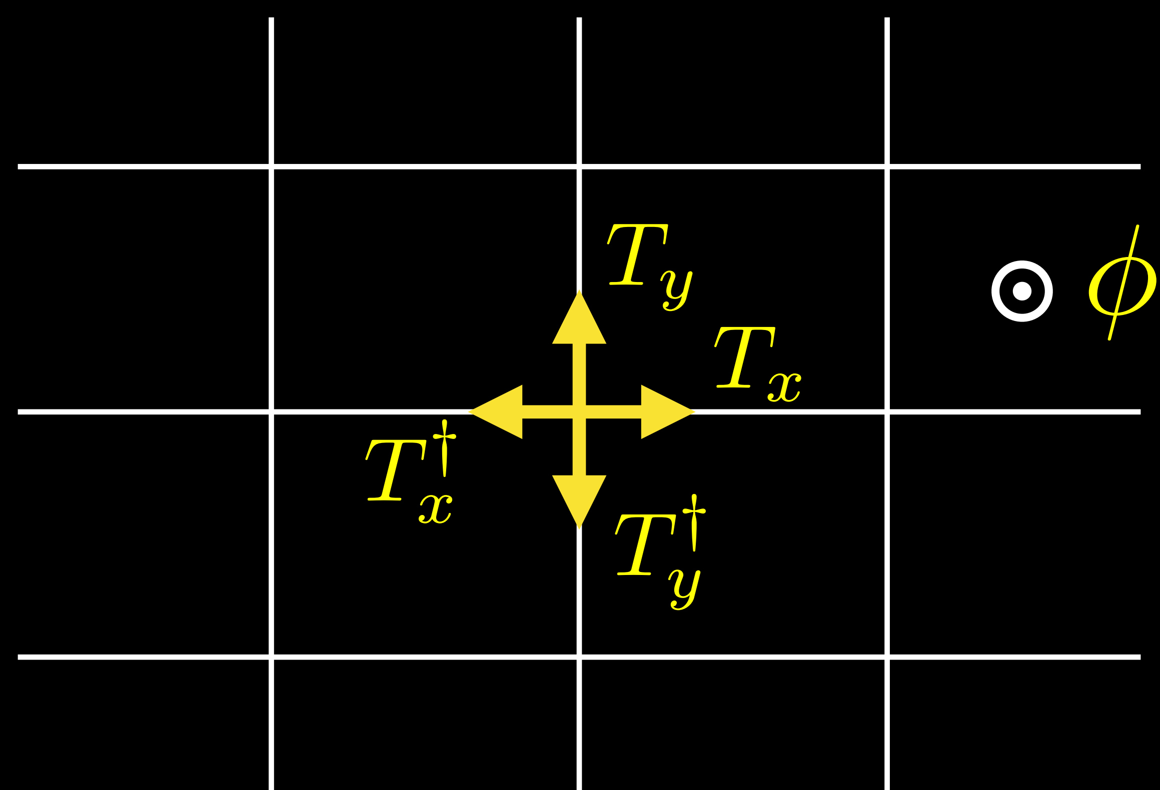
Hecke trans. $h^{\rightarrow} : \mathcal{L} \rightarrow \hat{\mathcal{L}}$

$$c_1(\hat{\mathcal{L}}) = c_1(\mathcal{L}) + 1$$

Duality of Quantum Group (Frenkel and Hernandez 2011)

Hamiltonian of Quantum Hall Effect

$$H = T_x + T_x^\dagger + T_y + T_y^\dagger$$



T_x T_y **:Representation of** $\mathcal{U}_q(sl_2)$ $\quad q = e^{2\pi i\phi}$

Duality of Quantum Group (Frenkel and Hernandez 2011)

ν_L :Landau Filling Factor

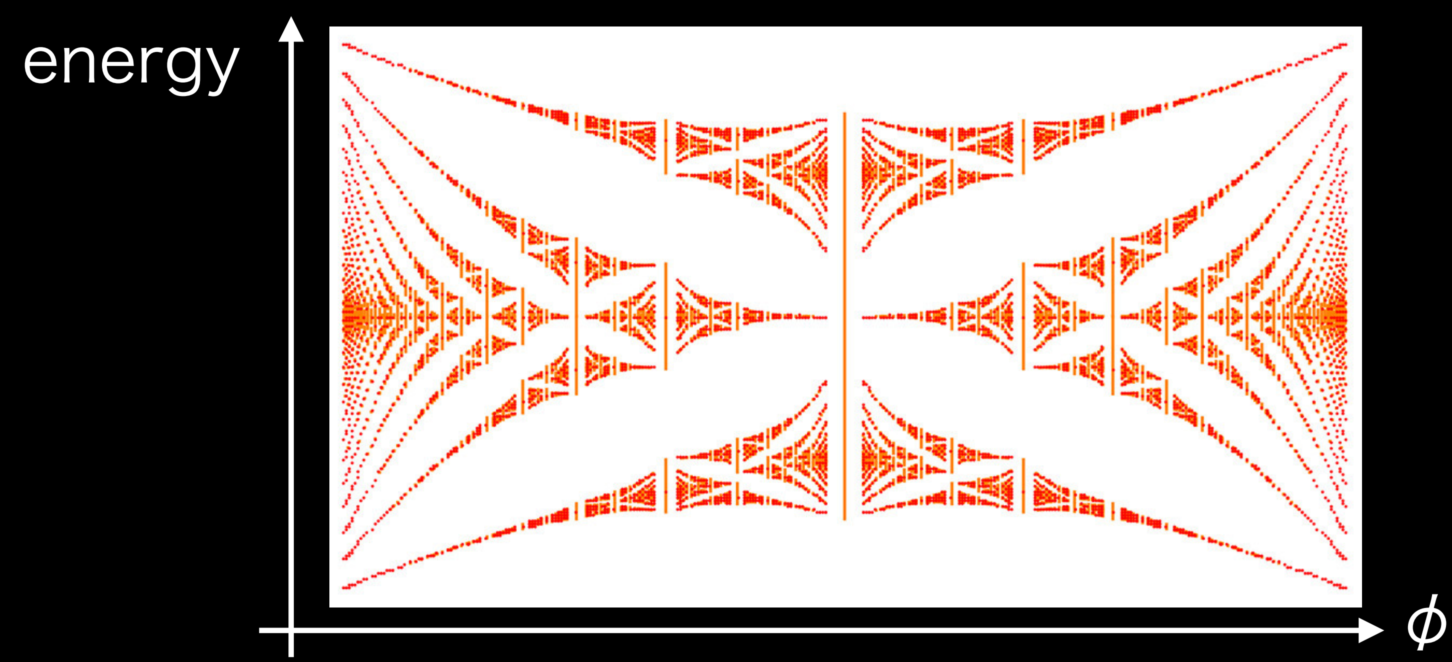
$$\nu_L = \phi \nu_B \qquad \nu_B = t\phi + s$$

Strong/Weak Duality $s \leftrightarrow t$

$$\left(\phi, \sigma_{xy} = \frac{e^2}{h} t\right) \leftrightarrow \left(1/\phi, \sigma_{xy} = \frac{e^2}{h} s\right)$$

Langlands Duality of Quantum Group

$$(\phi, \mathcal{U}_q(sl_2)) \leftrightarrow (1/\phi, \mathcal{U}_{{}^L q}(sl_2))$$



(String Theoretical Relation: Hatsuda, Katsura, Tachikawa 2016)

Home Messages

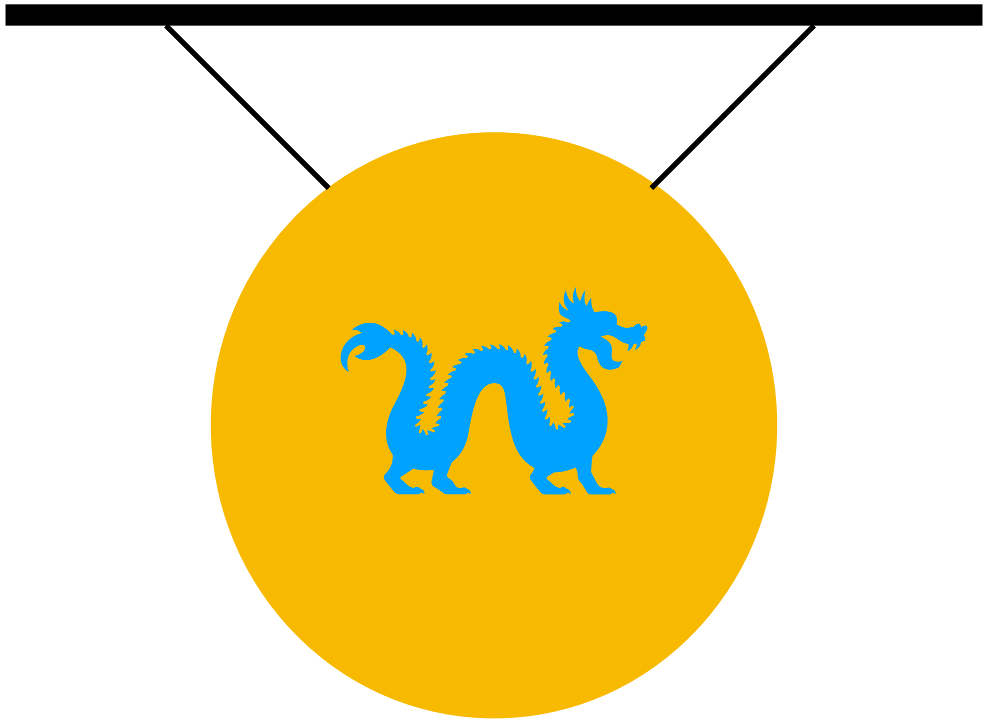
Symmetry Protected Topological (SPT) Matter would generally respect the Langlands Program

AZ class	space of projectors in momentum space	BL class	$N_f^{\min}$	fermionic replica NLσM target space	topological or WZW term
A	$\{ Q(k) \in G_{m,m+n}(\mathbb{C}) \}$	$\mathbf{0}$	1	$\mathrm{U}(2N)/\mathrm{U}(N)\times\mathrm{U}(N)$	Pruisken
AI	$\{ Q(k) \in G_{m,m+n}(\mathbb{C}) \mid Q(k)^* = Q(-k) \}$	$\mathbf{4}_+$	2	$\mathrm{Sp}(2N)/\mathrm{Sp}(N)\times\mathrm{Sp}(N)$	N/A
AII	$\{ Q(k) \in G_{2m,2(m+n)}(\mathbb{C}) \mid (i\sigma_y)Q(k)^*(-i\sigma_y) = Q(-k) \}$	$\mathbf{3}_+$	1	$\mathrm{O}(2N)/\mathrm{O}(N)\times\mathrm{O}(N)$	$\mathbb{Z}_2$
AIII	$\{ q(k) \in \mathrm{U}(m) \}$	**1** or **2**	1 or 2	$\mathrm{U}(N)\times\mathrm{U}(N)/\mathrm{U}(N)$	WZW
BDI	$\{ q(k) \in \mathrm{U}(m) \mid q(k)^* = q(-k) \}$	$\mathbf{9}_+$	2	$\mathrm{U}(2N)/\mathrm{Sp}(N)$	N/A
CII	$\{ q(k) \in \mathrm{U}(2m) \mid (i\sigma_y)q(k)^*(-i\sigma_y) = q(-k) \}$	$\mathbf{9}_-$	2	$\mathrm{U}(2N)/\mathrm{O}(2N)$	$\mathbb{Z}_2$
D	$\{ Q(k) \in G_{m,2m}(\mathbb{C}) \mid \tau_x Q(k)^* \tau_x = -Q(-k) \}$	$\mathbf{3}_-$	1	$\mathrm{O}(2N)/\mathrm{U}(N)$	Pruisken
C	$\{ Q(k) \in G_{m,2m}(\mathbb{C}) \mid \tau_y Q(k)^* \tau_y = -Q(-k) \}$	$\mathbf{4}_-$	2	$\mathrm{Sp}(N)/\mathrm{U}(N)$	Pruisken
DIII	$\{ q(k) \in \mathrm{U}(2m) \mid q(k)^T = -q(-k) \}$	**5** or **7**	1 or 2	$\mathrm{O}(2N)\times\mathrm{O}(2N)/\mathrm{O}(2N)$	WZW
CI	$\{ q(k) \in \mathrm{U}(m) \mid q(k)^T = q(-k) \}$	**6** or **8**	2 or 4	$\mathrm{Sp}(N)\times\mathrm{Sp}(N)/\mathrm{Sp}(N)$	WZW

Schnyder et al. (2009)

String → Langlands Program ← Condensed Matter

Kapustin and Witten (2005),
Gukov and Witten (2006),
Hausel (2002),
Frenkel and Gaiotto (2018)



5

SCFT/VOA correspondence via Ω-deformation

based on arXiv:1904.00927

Saebyeok Jeong

Gong Show, String Math 2019

C.N. Yang Institute for Theoretical Physics, Stony Brook University

*See also [Oh-Yagi '19], [Pan-Peelaers '19], [Dedushenko-Fluder '19].

VOA from (Q+S)-cohomology

Four-dimensional $\mathcal{N}=2$ superconformal algebra admits a fermionic generator of the form $\mathbb{Q} \equiv$ "$\mathcal{Q}+\mathcal{S}$", for which [Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees '13]

$$\mathbb{Q}^2 = 0, \quad [\mathbb{Q},\ L_{+1,0,-1}] = 0$$
$$\{\mathbb{Q},\ \widetilde{\mathcal{Q}}_{1\dot{-}}\} = \bar{L}_{-1} + \mathcal{R}^-, \quad \{\mathbb{Q},\ \mathcal{S}_2^-\} = \bar{L}_{+1} - \mathcal{R}^+, \quad \{\mathbb{Q},\ \mathbb{Q}^\dagger\} = 2(\bar{L}_0 - \mathcal{R}).$$

In particular, R-twisted anti-holomorphic conformal transformations on a plane are $\mathbb{Q}$-exact.

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In particular, R-twisted anti-holomorphic conformal transformations on a plane are $\mathbb{Q}$-exact.

Hence, from *Schur* operators located at the origin,

$$\{\mathbb{Q},\ \mathcal{O}(0)] = 0, \quad \mathcal{O}(0) \neq \{\mathbb{Q},\ \mathcal{O}'(0)],$$

we can build the *twisted-translations* of local operators

$$\mathcal{O}(z,\bar{z}) = e^{zL_{-1}+\bar{z}\hat{L}_{-1}}\mathcal{O}(0)e^{-zL_{-1}-\bar{z}\hat{L}_{-1}},$$

whose $\mathbb{Q}$-cohomology forms a vertex operator algebra.

VOA for Lagrangian $\mathcal{N} = 2$ superconformal theories

Free hypermultiplet gives symplectic bosons (also known as $\beta\gamma$ system).

Free vectormultiplet gives *bc* ghosts.

For interacting theories, the prescription is first to take the naive tensor product and then to pass to the cohomology with respect to the nilpotent BRST operator.

An alternative approach from Ω-deformation?

Ω-deformation of holomorphic-topological theory

For the $\mathcal{N}=2$ superconformal theory on $\mathcal{C}\times\mathcal{C}^{\perp}$, we can make the *holomorphic-topological* twist [Kapustin '06] between $U(1)_{\mathcal{C}}\times U(1)_{\mathcal{C}^{\perp}}$ and $U(1)_R\times U(1)_r\subset SU(2)_R\times U(1)_r$, with the scalar supercharge $\mathcal{Q}=\mathcal{Q}^1_-+\widetilde{\mathcal{Q}}^1_{\dot{-}}$,

$$\{\mathcal{Q},\mathcal{Q}^2_+\}=-\mathcal{P}_{+\dot{-}},\quad \{\mathcal{Q},\widetilde{\mathcal{Q}}^2_{\dot{+}}\}=\mathcal{P}_{-\dot{+}}$$
$$\{\mathcal{Q},\mathcal{Q}^2_-\}=-\{\mathcal{Q},\widetilde{\mathcal{Q}}^2_{\dot{-}}\}=-\mathcal{P}_{-\dot{-}}.$$

Hence as a $\mathcal{Q}$-cohomological field theory, the $\mathcal{N}=2$ superconformal theory is topological along $\mathcal{C}^{\perp}$ and holomorphic along $\mathcal{C}$.

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$$\{\mathcal{Q},\mathcal{Q}^2_{+}\}=-\mathcal{P}_{+\dot{-}},\quad \{\mathcal{Q},\widetilde{\mathcal{Q}}^2_{\dot{+}}\}=\mathcal{P}_{-\dot{+}}$$
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Hence as a $\mathcal{Q}$-cohomological field theory, the $\mathcal{N}=2$ superconformal theory is topological along $\mathcal{C}^{\perp}$ and holomorphic along $\mathcal{C}$.

The action for the vector multiplet is $\mathcal{Q}$-exact:

$$S_{\text{vec}}=\mathcal{Q}\left(\cdots\right).$$

The action for the hypermultiplet splits into $\mathcal{Q}$-closed part and $\mathcal{Q}$-exact part:

$$S_{\text{hyp}}=\int_{\mathcal{C}}d^2z\;\mathcal{O}^{(2)}_{\mathcal{W}}+\mathcal{Q}\left(\cdots\right),\quad \mathcal{W}=\int_{\mathcal{C}^{\perp}}Q_z(\partial_{\bar{z}}-i\mathcal{A}_{\bar{z}})\tilde{Q}$$

Ω-deformation of holomorphic-topological theory

As in [Nekrasov '18], [Costello-Yagi '18] , we can define a deformed supersymmetry generator $\mathcal{Q}_\varepsilon$ such that

$$\mathcal{Q}_\varepsilon^2 = \varepsilon(\mathcal{D}_{\mathcal{C}^\perp}\iota_V + \iota_V \mathcal{D}_{\mathcal{C}^\perp}) = \varepsilon\mathcal{L}_V + \mathsf{Gauge}[\varepsilon\iota_V\mathcal{A}],$$

where V is the isometry of $\mathcal{C}^\perp$. The Ω-deformed action can be obtained by replacing $\mathcal{Q}$ by $\mathcal{Q}_\varepsilon$.

Ω-deformation of holomorphic-topological theory

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The $\mathcal{Q}_\varepsilon$-variations of fermions give

$$\mathcal{F} = 0, \quad \iota_V F - iD_{\mathcal{C}^\perp}\iota_V\phi = 0, \quad \mathrm{D} = 0,$$
$$\mathcal{F}_{\bar{z}} + \varepsilon\iota_V \mathrm{D}_{\bar{z}} = 0, \quad \mathcal{D}_{\mathcal{C}^\perp} q_z + \varepsilon\iota_V h_z = 0, \quad \mathcal{D}_{\mathcal{C}^\perp}\tilde{q} + \varepsilon\iota_V h = 0.$$

By integrating out the auxiliary fields, fixing a gauge, and introducing polar coordinates on $\mathcal{C}^\perp$,

$$\partial_t A_{\bar{z}} = -\frac{1}{2\bar{\varepsilon}} q_{\bar{z}}\tilde{q}^\dagger, \quad \partial_t q_z = -\frac{i}{2\bar{\varepsilon}} D_z\tilde{q}^\dagger, \quad \partial_t\tilde{q} = \frac{i}{2\bar{\varepsilon}} D_z q_{\bar{z}},$$

This is the gradient flow generated by $\mathrm{Re}\left(\frac{1}{\varepsilon}\int_{\mathcal{C}} q_z D_{\bar{z}}\tilde{q}\right) = \mathrm{Re}\left(\frac{\mathcal{W}}{\varepsilon}\right)$.

Localization and VOA

Due to the convergence of the action, $A_{\bar{z}}$, q_z, and $\tilde{q}$ should end on the critical points $\{d\mathcal{W} = 0\}$ as $t \to \infty$. The remaining two-dimensional path integral is defined on the gradient flows emanating from those critical points, i.e., Lefschetz thimbles.

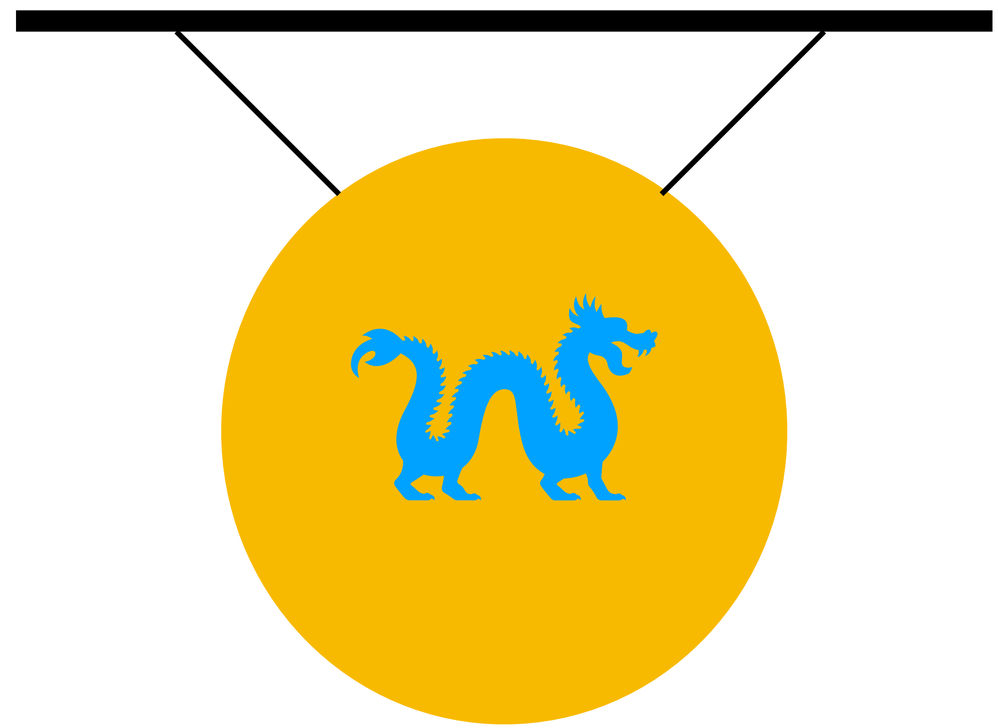
The action for this two-dimensional path integral is obtained as

$$S = \frac{1}{\varepsilon} \int_{\mathcal{C}} d^2 z \, q_z D_{\bar{z}} \tilde{q}.$$

By further fixing the remnant gauge by $A_{\bar{z}} = 0$, we arrive at

$$\frac{1}{\varepsilon} \int_{\mathcal{C}} \left(\operatorname{Tr} b \bar{\partial} c + \sum_i q^i \bar{\partial} \tilde{q}^i \right).$$

This is the action for the two-dimensional bc-$\beta\gamma$ system. The algebra of local operators of this theory recovers the VOA that we wanted.



6

Modular Graph Functions in Physics

Justin Kaidi

String Math 2019
July 1, 2019

UCLA

[1608.04393] **JK**, D'Hoker
[1809.05122] **JK**, Gerken
[1902.04180] **JK**, D'Hoker

Genus one 4-graviton scattering

- **Consider 4 graviton scattering at one loop,**

$$\mathcal{A}_1^{(4)} = 2\pi\mathcal{R}^4 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \mathcal{B}_4(s_{ij};\tau) \qquad s_{ij} = -\frac{\alpha'}{4}(k_i + k_j)^2$$

–Partial amplitudes $\mathcal{B}_4(s,t,u;\tau)$ are integrals over vertex operator insertions,

$$\mathcal{B}_4(s_{ij}) = \prod_{i=1}^{4} \int_{\Sigma} \frac{d^2 z_i}{\tau_2} \exp\left\{ \sum_{1\leq i<j\leq 4} s_{ij} G(z_i - z_j|\tau) \right\}$$

– We work with the Arakelov Greens function,

$$\partial_{\bar{z}}\partial_z G(z|\tau) = -\pi\delta^{(2)}(z) + \frac{\pi}{\tau_2} \qquad \int_{\Sigma} dz\, G(z|\tau) = 0$$

– Admits a Kronecker-Eisenstein series representation,

$$G(z|\tau) = \sum_{p\in\Lambda}{}' \frac{\tau_2}{\pi|p|^2} e^{2\pi i(n\alpha - m\beta)} \qquad \begin{array}{l} p = m + n\tau \\ z = \alpha + \beta\tau \end{array}$$

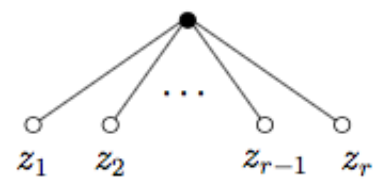
Diagrammatic Expansion

- **We now try to calculate the partial amplitudes in α'-expansion,**

$$\mathcal{B}_4(s_{ij}) = \sum_{w=0}^{\infty} \frac{1}{w!} \prod_{i=1}^{4} \int_\Sigma \frac{d^2 z_i}{\tau_2} \left(\sum_{1\leq i<j\leq 4} s_{ij} G(z_i - z_j|\tau) \right)^w$$

- **Graphical notation is useful** ['15 D'Hoker, Green, Vanhove]**,**

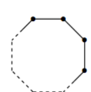
$= G(z_i - z_j|\tau)$



$$= \int_\Sigma \frac{d^2 z}{\tau_2} \prod_{i=1}^{r} G(z - z_i|\tau)$$

- **Given any graph Γ, we have** $\mathcal{C}_\Gamma(\tau) = \left(\prod_{k=1}^m \int_\Sigma \frac{d^2 z_k}{\tau_2} \right) \prod_{1\leq i<j\leq m} G(z_i - z_j|\tau)^{\nu_{ij}}$**,**

$$\mathcal{C}_\Gamma(\tau) = \sum_{p_1,\dots,p_w \in \Lambda}{}' \left(\prod_{r=1}^{w} \frac{\tau_2}{\pi |p_r|^2} \right) \prod_{i=1}^{m} \delta \left(\sum_{r=1}^{w} \Gamma_{ir} p_r \right) \qquad \text{“MGF”}$$

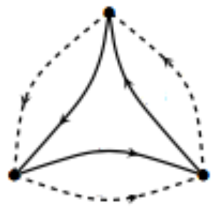
  - Example 1: $E_w \equiv \mathcal{C}\Big[$  $\Big] = \sum'_{p\in\Lambda} \left(\frac{\tau_2}{\pi |p|^2} \right)^w$
  - Example 2: $C_{1,1,1} \equiv \mathcal{C}\Big[$  $\Big] = \sum'_{p_1,p_2,p_3\in\Lambda} \frac{(\tau_2/\pi)^3}{|p_1|^2 |p_2|^2 |p_3|^2} \delta(p_1 + p_2 + p_2)$

Relations between MGFs

- **Modular graph functions obey a rich set of identities.**
  - Algebraic identities, e.g.

$$\begin{aligned} C_{1,1,1} &= E_3 + \zeta(3) \\ 7C_{2,2,2,1} &= 21E_4E_3 + 14C_{3,2,2} + 28C_{421} - 31E_7 \end{aligned}$$

  - All such algebraic identities up to weight 7 obtained via sieve algorithm in ['16 **JK**, D'Hoker].
  - Special class of identities: "holomorphic subgraph reduction" ['18 **JK**, Gerken]



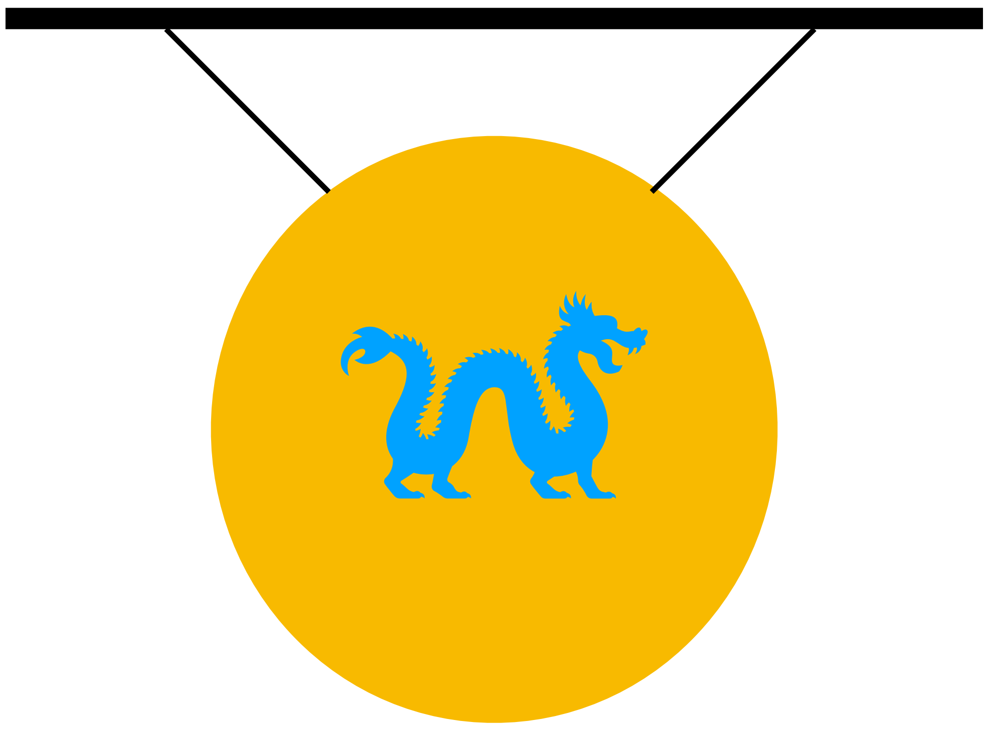
  - Differential identities, e.g. ['16 **JK**, D'Hoker]

$$(\Delta - w(w-1))E_w = 0 \qquad (\Delta - 2)C_{2,1,1} = 9E_4 + E_2^2$$

  - Fourier and Poincaré series obtained in ['18 D'Hoker, Duke; '19 **JK**, D'Hoker]. This enables integration of MGFs to get full string amplitudes!

The End (for now)

Thank you!



7

Super J-holomorphic curves

Enno Keßler
joint with A. Sheshmani and S.-T. Yau

Center of Mathematical Sciences and Applications, Harvard University
DFG Research Fellow

Gong Show StringMath 2019
1st July 2019

Definition

A super Riemann surface is a complex supermanifold M of dimension $1|1$ together with a holomorphic distribution $\mathcal{D} \subset TM$ such that the commutator of vector fields induces an isomorphism $\mathcal{D} \otimes \mathcal{D} \to {}^{TM}\!/_{\mathcal{D}}$.

Theorem (EK[1], see also Howe 1979)

Let $i\colon |M| \to M$ be a map from a $2|0$-dimensional manifold into a $2|2$-dimensional supermanifold which restricts to the identity of topological manifolds. A super Riemann surface structure is equivalent to a Riemannian metric g, a spinor bundle S and a gravitino $\chi \in \Gamma\left(T^\vee|M| \otimes S\right)$ on $|M|$ (up to Weyl- and super Weyl transformations).

[1] Keßler (2019). *Supergeometry, Super Riemann Surfaces and the Superconformal Action Functional*. Springer LNM 2230, to appear

Definition

Let I be the almost complex structure on M and N a symplectic manifold with compatible almost complex structure J. For $\Phi\colon M \to N$, define the operator $\overline{D}_J\Phi \in \Gamma\left(\mathcal{D}^\vee \otimes \Phi^* TN\right)^{0,1}$ by

$$\overline{D}_J\Phi = \frac{1}{2}\left(d\Phi + J \circ d\Phi \circ \mathrm{I}\right)\big|_{\mathcal{D}}.$$

We will call maps Φ such that $\overline{D}_J\Phi = 0$ super J-holomorphic curves.

- If the almost complex structure J is integrable the map Φ is holomorphic.
- If Φ is a super J-holomorphic curve it is a critical point of the superconformal action on M, or "spinning string".

For $i\colon |M| \to M$ and $\Phi\colon M \to N$ define the component fields

$$\begin{aligned}
\varphi &= \Phi \circ i\colon |M| \to N, \\
\psi &= i^* \, d\Phi|_{\mathcal{D}} \in \Gamma\left(S^\vee \otimes \varphi^* TN\right), \\
F &= i^* \Delta^{\mathcal{D}} \Phi \in \Gamma\left(\varphi^* TN\right).
\end{aligned}$$

- In good coordinates (x^a, η^α) on M:
  $\Phi(x, \eta) = \varphi(x) + \eta^\mu{}_\mu\psi(x) + \eta^3\eta^4 F(x)$
- There is a supermanifold structure on $\mathrm{Hom}(M, N)$ given by the exponential map and charts around (φ_0, ψ_0, F_0) given by

$$\Gamma\left(\varphi_0^* TN\right) \oplus \Gamma\left(S^\vee \otimes \varphi_0^* TN\right) \oplus \Gamma\left(\varphi_0^* TN\right).$$

Moduli space of super J-holomorphic curves

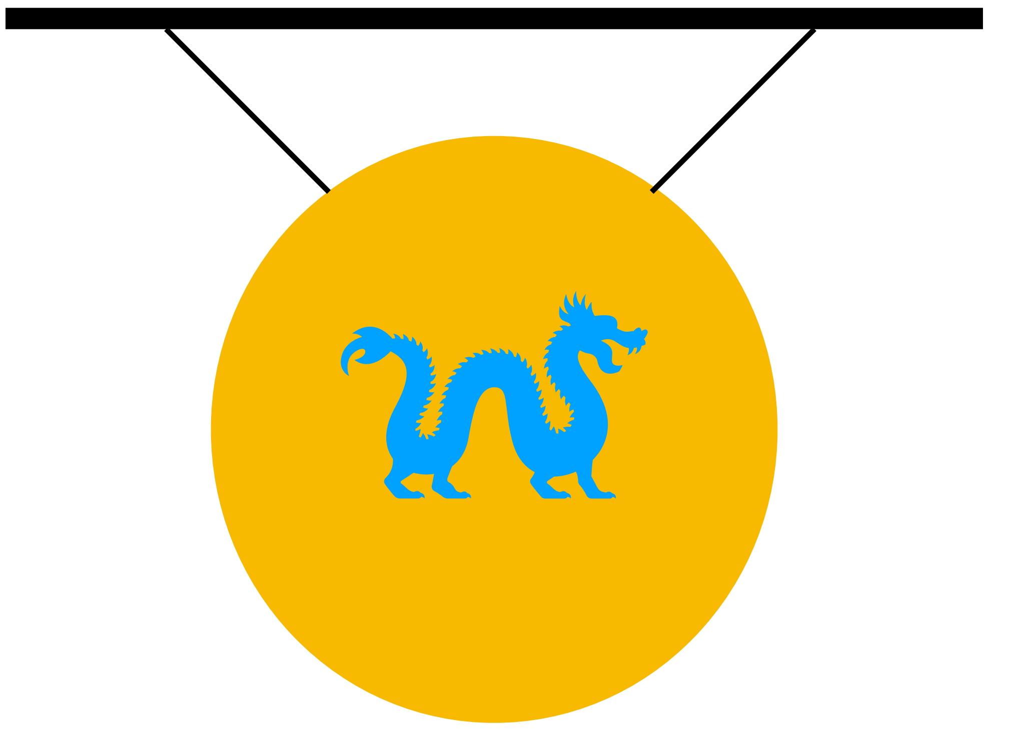
$\Phi\colon M \to N$ is a super J-holomorphic curve if and only if in component fields

$$0 = \psi + \mathrm{I} \otimes J\psi, \qquad 0 = F \in \Gamma\left(\varphi^* TN\right),$$
$$0 = \overline{\partial}_J\varphi + \langle Q\chi, \psi\rangle + \frac{1}{4}\operatorname{Tr}_{g_S^\vee}\langle\psi, \varphi^*\nabla J\rangle\,\gamma\,\mathrm{I}\,\psi,$$
$$0 = \not{D}^{1,0}\psi - (1 + \mathrm{I}\otimes J)\left(2\langle\vee Q\chi, d\varphi\rangle - \frac{1}{3}SR^N(\psi)\right).$$

- The moduli space of super J-holomorphic curves should obtain a subsupermanifold structure from $\operatorname{Hom}(M, N)$.
- The expected real dimension of the moduli space is

$$\begin{aligned}&\operatorname{ind}\overline{\partial}_J|\operatorname{ind}\not{D}^{1,0}\\ &\quad= 2n(1-p) + 2\langle c_1(TN), A\rangle\,|2(n-1)(1-p) + 2\langle c_1(TN), A\rangle\,.\end{aligned}$$

where p is the genus of M, $2n$ is the real dimension of N and $A = [\operatorname{im}\varphi] \in H_2(N)$.



8

Fourier-Mukai Transforms of Slope Semistable Sheaves on Weierstrass Elliptic Surfaces

Wanmin Liu (Uppsala Univ.)
Jason Lo (California State Univ. Northridge)

STRING MATH 2019

01-05 JULY 2019, UPPSALA, SWEDEN

Thanks to **IBS Center for Geometry and Physics** in Pohang, South Korea
Liu was supported by IBS-R003-D1
Preprint is available at wanminliu.github.io

MOTIVATION

X smooth proj var/ $\mathbb{C}$

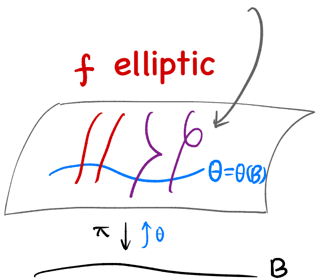
$D^b(X)$ bounded derived category of coherent sheaves

$\Phi \in \mathrm{Aut}(D^b(X))$

(Mumford slope, Gieseker, PT, Bridgeland etc)

$$\begin{array}{ccc} \sigma\text{- stability condition} & \rightsquigarrow & \sigma\text{-invariant} \\ \big\downarrow \Phi & & \Phi \,\big\downarrow \quad \text{relation/ symmetry/ modularity ?} \\ \sigma' = \Phi(\sigma) & \rightsquigarrow & \sigma'\text{- invariant} \end{array}$$

QUESTION



$$\begin{array}{c} X \\ \pi \big\downarrow \big\uparrow \theta \\ B \end{array}$$

elliptic fiberation with a section θ $\qquad \Theta^2 = -e$

over a smooth base

Φ relative Fourier-Mukai transform $\in \mathrm{Aut}(D^b(X))$ $\qquad \Phi[1] \circ \hat{\Phi} = \hat{\Phi} \circ \Phi[1] = \mathrm{id}_{D^b(X)}$

$\mu_{\bar{\omega}}$ slope stability condition $\left(\mu_{\bar{\omega}}(-) = \dfrac{c_1(-) \cdot \bar{\omega}}{\mathrm{rank}(-)},\ \mathrm{Coh}(X) \right)$

$$\bar{\omega} = \underbrace{(\Theta + mf)}_{\text{ample}} + \alpha \underbrace{f}_{\text{nef}}$$

α: fixed positive number

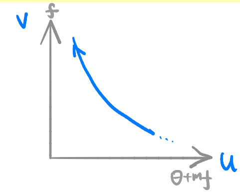
What is a notion of stability condition for slope stability condition under $\Phi[1]$?

Key Premise: we do NOT fix Chern characters
(Otherwise, lots of work by Bruzzo, Maciocia, Yoshioka and many...)

Limit Bridgeland Stability Condition σ^ℓ

$$\omega = u(\theta + mf) + vf$$

along CURVE $u^2 + uv = \alpha + m - e$ ☆



$$\mathrm{Coh}(X) \xrightarrow{(\mathcal{T}_\omega,\ \mathcal{F}_\omega)}$$

$$\mathcal{B}^\ell \xleftarrow[\text{as } v\to\infty]{\text{limit along ☆}} \mathcal{B}_\omega = \langle \mathcal{F}_\omega[1], \mathcal{T}_\omega \rangle$$

$$\sigma_\omega = \left(Z_\omega := -\int_X e^{-i\omega} ch(-),\ \mathcal{B}_\omega \right)$$

Bridgeland stab condition

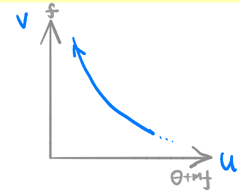
$$\boxed{\sigma^\ell = (Z^\ell, \mathcal{B}^\ell) := \lim_{\substack{v \to +\infty \\ \text{along ☆}}} \sigma_\omega}$$

Limit Bridgeland Stability Condition σ^ℓ

$$
\begin{tikzcd}
D^b(X) & & D^b(X) \\
\mathrm{Coh}(X) \arrow[rr, shift left, "\Phi[1]\ \cong"] \arrow[ddrr, shift left, "\text{Thm (A)}"] & & \Phi(\mathrm{Coh}(X))[1] \arrow[ll, shift left, "\hat{\Phi}"] \\
& & \\
& & \mathcal{B}^\ell \arrow[uull, shift left, "\text{Thm (B)}"] \arrow[uu, "\not"]
\end{tikzcd}
$$

$\omega = u(\theta + mf) + vf$

along CURVE $u^2 + uv = \alpha + m - e$ ☆



$$
\begin{tikzcd}
 & \mathrm{Coh}(X) \arrow[d, "(\mathcal{T}_\omega,\ \mathcal{F}_\omega)"] \\
\mathcal{B}^\ell & \mathcal{B}_\omega = \langle \mathcal{F}_\omega[1], \mathcal{T}_\omega \rangle \arrow[l, "\text{limit along ☆}"', "\text{as } v\to\infty"]
\end{tikzcd}
$$

$\sigma_\omega = \left(Z_\omega := -\int_X e^{-i\omega} ch(-),\ \mathcal{B}_\omega \right)$

Bridgeland stab condition

$$\boxed{\sigma^\ell = (Z^\ell, \mathcal{B}^\ell) := \lim_{\substack{v \to +\infty \\ \text{along ☆}}} \sigma_\omega}$$

Theorem (L-Lo)

$$\begin{tikzcd} \mathrm{Coh}(X) \arrow[r, shift left, "\Phi[1]\ \simeq"] \arrow[dr, shift left, "\text{Thm (A)}"] & \Phi(\mathrm{Coh}(X))[1] \arrow[l, shift left, "\hat{\Phi}"] \\ & \mathcal{B}^l \arrow[ul, shift left, "\text{Thm (B)}"] \arrow[u, "/"] \end{tikzcd}$$

X Weierstrass elliptic surface

(A) $(\mu_{\bar{\omega}}\text{-stability}, \mathrm{Coh}(X)) \xrightarrow{\Phi[1]} (Z^l\text{- stability}, \mathcal{B}^l)$

E - $\mu_{\bar{\omega}}$ stable $\qquad$ $F := \Phi(E)[1]$ Z^l- stable

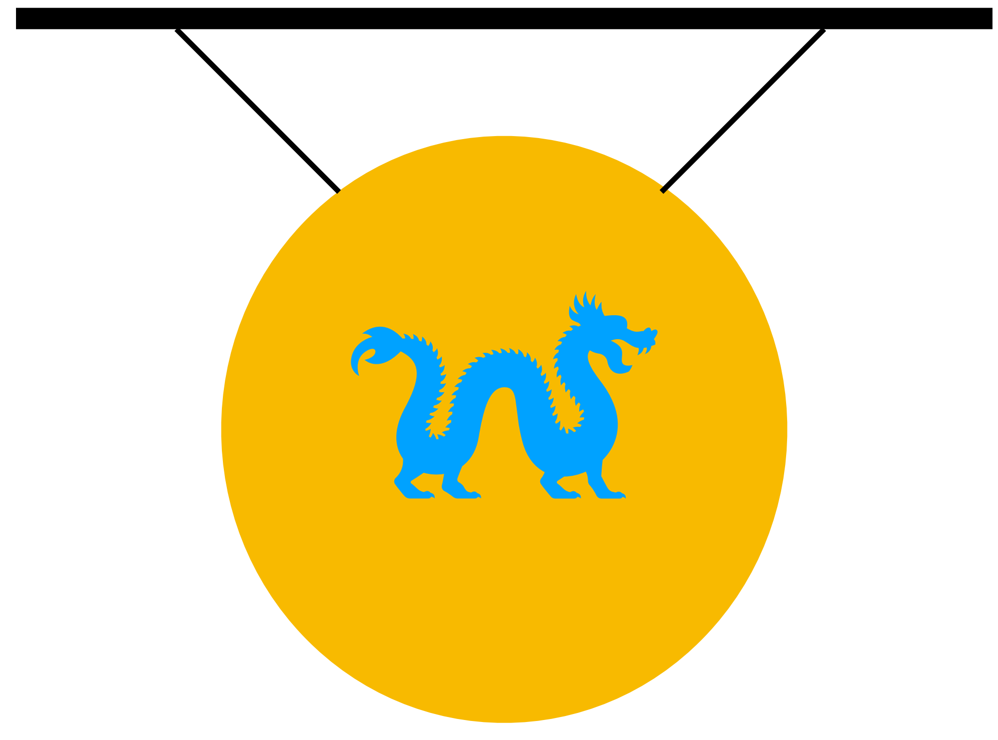
$\downarrow \qquad\qquad \downarrow$

(B) $(\mu_{\bar{\omega}}\text{-semistability}, \mathrm{Coh}(X)) \xleftarrow{\hat{\Phi}} (Z^l\text{-semistability}, \mathcal{B}^l)$

small modification

$\hat{\Phi}(F')$ - $\mu_{\bar{\omega}}$ - s.s. $\qquad$ $\exists \quad 0 \to F' \to F \to F'' \to 0 \quad$ in $\mathcal{B}^l$

$\hat{\Phi}(F'')$ torsion sheaf



9

Deriving on-shell open string field amplitudes

without using the Feynman rule

Toru Masuda

CEICO, Institute of Physics, Czech Academy of Sciences, Prague

String-Math 2019, Uppsala, Sweden, 1st July, 2019.

(First Slide)

I would like to thank the organizers for giving me this opportunity to introduce my recent research, and also thank you all for coming to this session. It is a great honor to be able to speak to you today.

My talk subject is related to open string field theory and how to derive on-shell scattering amplitudes in this framework, but it's not a very familiar topic of the recent String-Math Conferences. So, let me first explain "what is the open string field theory" briefly.

Advertisement

this talk is based on a collaborative work with H. Matsunaga, arXiv:1907.***** (coming soon).

(Second Slide)

Open string field theory (OSFT) is a field-theoretic formulation of open string theory. Today, we consider Witten's bosonic OSFT whose action has a close resemblance to the 3-dim. Chern-Simons theory:

$$S[\Phi] = -\frac{1}{2}\int \Phi * Q_B \Phi - \frac{1}{3}\int \Phi * \Phi * \Phi$$

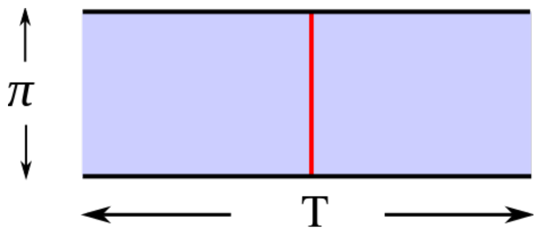
$$Q_B : \mathcal{H} \to \mathcal{H}, \quad * : \mathcal{H} \times \mathcal{H} \to \mathcal{H}, \quad \int : \mathcal{H} \to \mathbb{R}.$$

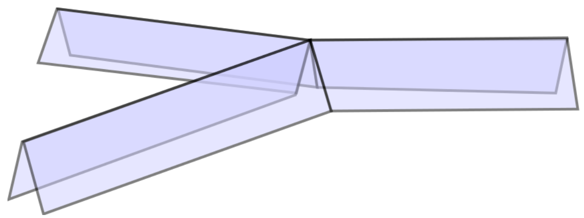
Witten's open SFT	3dim. Chern-Simons theory
$*$	$\wedge$
Q_B	d
ghost number	rank of the differential form

Algebraic structure looks almost the same; but objects with *negative ghost number* make OSFT dynamical (and interesting in a sense).

(Third Slide)

As usual (local) quantum field theory, we can compute the scattering amplitudes in OSFT using the Feynman rule; yet, it might not look ”very natural” to decompose the world-sheet into vertices and propagators.





So, *let us look for another way of calculation.* (1) Since the scattering amplitude is an observable (= a physical quantity), it should be gauge invariant. (2) We also know that a BRST exact state drops from the on-shell amplitude. These two conditions are key to find the new formula for the scattering amplitudes.

(Fourth Slide)

Then, we use a classical solution Ψ, a tachyon vacuum solution Ψ_T, a set of external states $\{\mathcal{O}_j\}$ as input of the our formula.

$$\begin{array}{ll} \Psi & \longrightarrow \\ \Psi_T & \longrightarrow \boxed{\text{our formula}} = (\text{on-shell amplitude}) \\ \{\mathcal{O}_j\} & \longrightarrow \end{array}$$

- The classical solution Ψ specify the D-brane configuration which we would like to consider.
- The tachyon vacuum solution Ψ_T is just a reference. The formula should be independent of the choice of Ψ_T.
- The external states $\mathcal{O}_j$ satisfy the on-shell condition

$$Q_\Psi \mathcal{O}_j = 0, \quad \text{the ghost number of } \mathcal{O}_j \text{ is } 1.$$

(Fifth Slide) Our formula for the 4pt amplitude is like this:

$$I_\Psi^{(4)}(\{\mathcal{O}_j\}) = \sum_{\text{permutation}} \int W_\Psi \mathcal{O}_{\sigma_1} W_\Psi \mathcal{O}_{\sigma_3} W_\Psi \mathcal{O}_{\sigma_3} A \mathcal{O}_{\sigma_4},$$

where $*$-symbol is omitted, and

(and we observed similar extension to N-point amplitude.)

$$W_\Psi = (\Psi - \Psi_T) * A_T + A_T * (\Psi - \Psi_T)$$

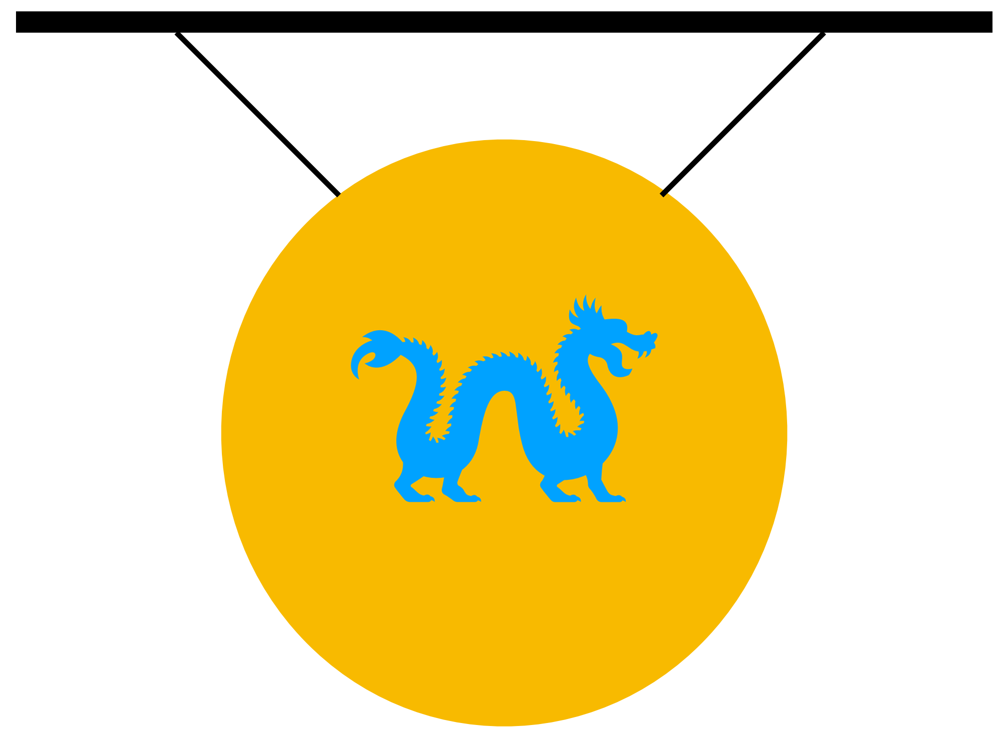
$$A = A_T - A_\Psi, \quad \text{s.t.} \quad Q_T A_T = 1, \quad Q_\Psi A_\Psi = 1.$$

This $I_\Psi^{(N)}$ has the following symmetry:

1. space-time gauge symmetry
2. decoupling of the null states ($\sim$ the BRST symmetry)
3. replacement of the reference Ψ_T
4. change the choice of A_T or A_Ψ

We checked this reproduces on-shell amplitudes for Ψ =EM solution, which is believed to express any (static) D-brane configurations.

If you are interested, please check our forthcoming paper. Thank you for your attention.



10

Partition Functions of $\mathcal{N} = 1$ Gauge Theories on $S^2 \times \mathbb{R}^2_\varepsilon$ and Dualities

Jun Nian

Leinweber Center for Theoretical Physics
University of Michigan

String Math 2019 Gong Show
Uppsala, July 1st, 2019

Based on 1812.11188 with Kimura, Zhao
and work in progress

Results of $Z_{\text{4D}\,\mathcal{N}=1}$

- $S^1 \times S^3$ ('13 Closset, Shamir)
- $S^1 \times \mathcal{M}_3$ ('14 Nishioka, Yaakov)
- S^4:
  - Technical Difficulty:
    ('11 Festuccia, Seiberg; '14 Knodel, Liu, Zayas; '15 Terashima)
  - $\mathcal{N}=1$ partition function on S^4 is unphysical.
    ('14 Gomis, Komargodski)
  - Analytic Continuation of Dimensions:
    ('17 Gorantis, Minahan, Naseer)
- $\mathbb{R}^2_\epsilon \times T^2$ ('15 Fujimori, Kimura, Nitta, Ohashi)
- $S^2 \times \mathbb{R}^2_\epsilon$:

$$ds^2 = \ell^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + |dw - iw\ell\varepsilon\, d\varphi|^2$$

$\mathcal{N}=1$ Localization

Killing spinor equation: ('12 & '13 Kawano, Matsumiya; '13 Lee, Yamazaki)

$$D_\mu \Upsilon = \frac{1}{2}\Gamma_\mu \Gamma_5 \Upsilon\,, \quad D_a \Upsilon = 0\,, \quad \mu \in \{1,2\}\,, \quad a \in \{3,4\}$$

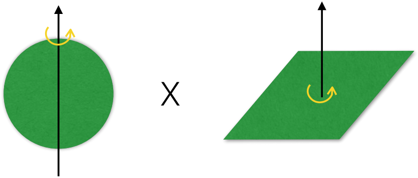
$$\Upsilon = \epsilon \otimes \zeta_+ + \tilde{\epsilon} \otimes \zeta_-\,, \quad \zeta_+ = (1,0)^T\,, \quad \zeta_- = (0,1)^T$$

Lagrangian (Higgs-branch localization '12 Benini, Cremonesi):

$$\mathscr{L}_{\text{exact}} = \delta\,\mathcal{V}_{\text{gauge}} + \delta\,\mathcal{V}_{\text{chiral}} + \delta\,\mathcal{V}_{\text{H}}$$

$$\text{with} \quad \mathcal{V}_H = \frac{i}{2}\left[\Sigma^\dagger \Gamma_5 \Xi + \Xi^\dagger \Gamma_5 \widetilde{\Sigma}\right](\bar{\Phi}^I \Phi^I - \eta)$$

(Anti-)Vortex at the north (south) pole and the origin:



$Z_{\text{4D}\,\mathcal{N}=1}$ on $S^2 \times \mathbb{R}^2_\epsilon$

$$Z_{l_i,\,\vec{\mu}_i} = \sum_{m_i, n_i} Z_{\text{class}}\, Z^{\text{vec}}_{\text{1-loop}}\, Z^{\text{chiral}}_{\text{1-loop}}\,, \qquad Z_{\text{class}} = \prod_{i=1}^{N} e^{-2\pi i (m_i - n_i)\tau_0}$$

$$Z^{\text{vec}}_{\text{1-loop}} = \prod_{\substack{i,j=1\\ i\neq j}}^{N} x^{-\frac{|(m_i-n_i)-(m_j-n_j)|}{2}} \left[1 - e^{\frac{2\pi i}{\varepsilon}(\widetilde{m}_{l_i}-\widetilde{m}_{l_j})} x^{|(m_i-n_i)-(m_j-n_j)|-(m_i+n_i)+(m_j+n_j)}\right]$$

$$Z^{\text{chiral}}_{\text{1-loop}} = \left[\prod_{i=1}^{N}\prod_{\substack{J=1\\ J\neq l_i}}^{N_F} \left(x^{1+n_i+m_i+2\ell(\widetilde{m}_J-\widetilde{m}_{l_i})}\right)^{\frac{\rho(n)-\rho(m)}{2}} \frac{\left(x^{2+2m_i+2\ell(\widetilde{m}_J-\widetilde{m}_{l_i})};\, x^2\right)_\infty}{\left(x^{-2m_i-2\ell(\widetilde{m}_J-\widetilde{m}_{l_i})};\, x^2\right)_\infty}\right]$$

$$\cdot \left[\prod_{i=1}^{N} \frac{1}{\prod_{q=0}^{m_i-1} \sinh\left[\frac{i\pi}{\ell\varepsilon}(q-m_i)\right]}\right]$$

Relation with $\mathcal{N}=2$ Nekrasov partition functions ('02 Nekrasov):

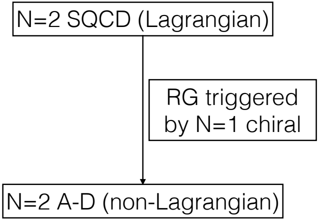
$$Z_\Omega^{\mathcal{N}=1\ \text{adj}} \cdot Z_\Omega^{\mathcal{N}=1\ \text{vec}} = Z_\Omega^{\mathcal{N}=2\ \text{vec}}\,, \qquad Z_\Omega^{\mathcal{N}=1\ \text{fun (anti-)chiral}} = Z_\Omega^{\mathcal{N}=2\ \text{fun hyper}}$$

Possible Applications

- Test Seiberg Duality ('95 Seiberg; '97 Elitzur, Giveon, Kutasov):

$$Z_{U(N)}(g_{YM},\, \vartheta,\, \widetilde{m}_i) = Z_{U(N_F-N)}(g^D_{YM},\, \vartheta^D\,,\, \widetilde{m}^D_i)$$

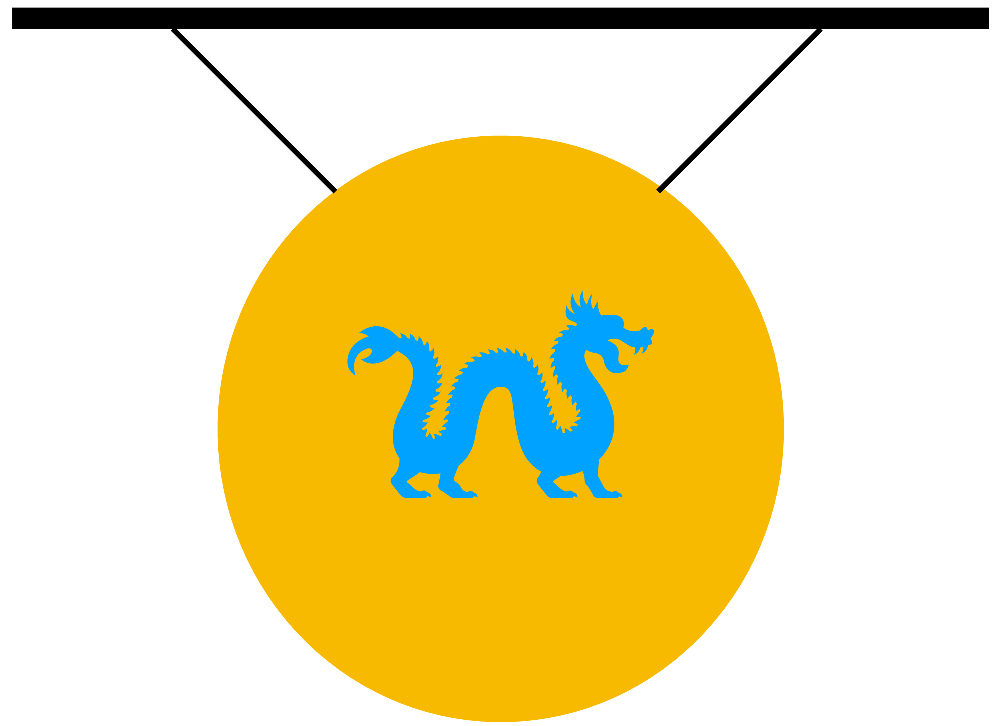
- $\mathcal{N}=2$ AGT for non-Lagrangian CFT ('16 Maruyoshi, Song):



(Work in progress)

- $\mathcal{N}=1$ AGT ('17 Mitev, Pomoni)
- $\mathcal{N}=1$ instanton counting via 6d $\mathcal{N}=(1,0) \overset{Flux}{\Longrightarrow}$ 4d $\mathcal{N}=1$?

  ('16 Razamat, Vafa, Zafrir; '17 Bah, Hanany, Maruyoshi, Razamat, Tachikawa, Zafrir)



11

Modularity from Monodromy

String Math 2019 - Gong Show

Thorsten Schimannek

based on [1902.08215], T.S.

and [190x.xxx], C. F. Cota, A. Klemm, T.S.

1.7.2019

Topological string partition function on elliptic/genus one fibered Calabi-Yau $X \to B$

dual via F-theory

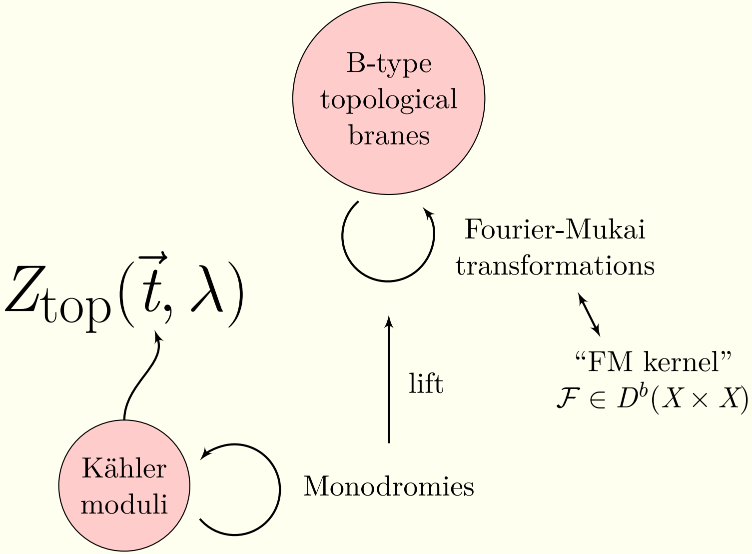
[Klemm,Mayr,Vafa'96],...

Elliptic genera of strings from D3-branes wrapping curves in B

Exhibit ***modular properties!***

QUESTION: Can we understand the modularity directly within topological string theory on X?

(Goes back to **[Candelas,Font,Katz,Morrison'94]***!)*



- We identified FM-kernels that act like $\Gamma_0(N)$ for elliptic and genus-one fibrations with reducible fibers

Elliptic fibrations with reducible fibers

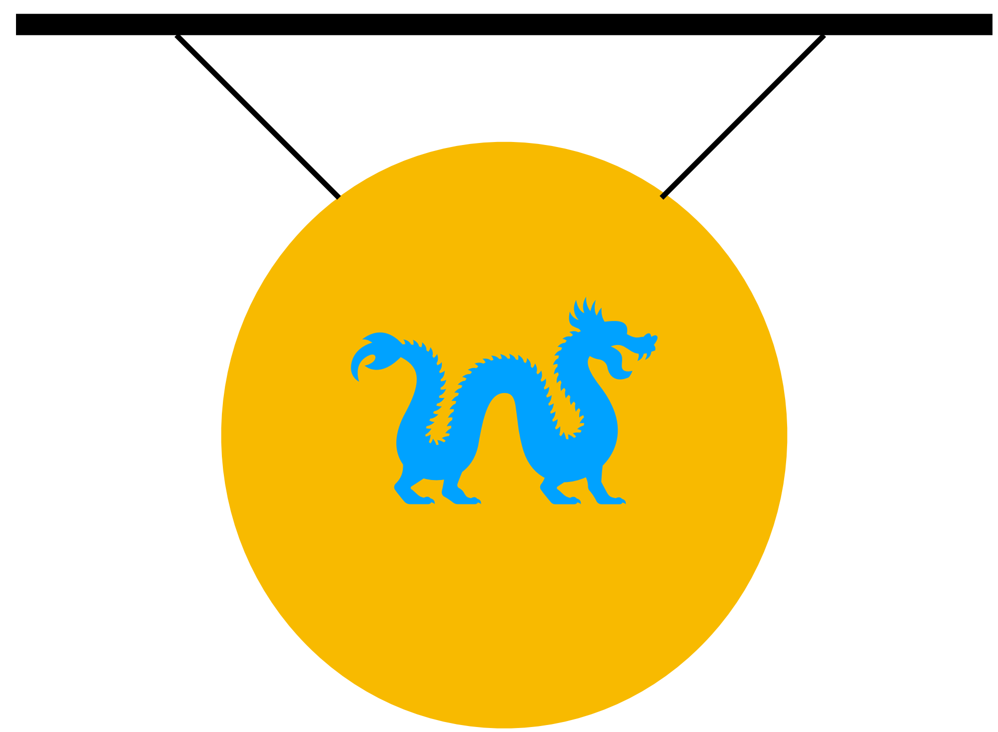
- Wave function interpretation of Z_{top} then implies elliptic transformation law w.r.t. volumes of fibral curves

$$Z_\beta(\tau, m_1, ..., m_a + \kappa\tau + \rho, ..., m_{\text{rk}(G)}, \lambda)$$
$$= \exp\left[-\frac{\beta_i}{2}\left(C^i_{aa}\kappa^2\tau + C^i_{(ab)}\kappa m^b\right)\right] Z_\beta(\tau, \vec{m}, \lambda)\,,$$

Genus-one fibrations *(i.e. no section)*

- New expressions in terms of Jacobi forms
- We study corresponding "E"-strings

Thank you for your attention!



12

Localizing Schur correlation functions

Yiwen Pan

JHEP 1802 (2018) 138, arXiv:1903.03623, work in progress

with Wolfger Peelaers

String-Math, Gong show, 2019 July

Background [Chris Beem's talk]

- Important progresses
- Beautiful correspondence: 4d $\mathcal{N} = 2$ SCFT and 2d VOA
  [Beem, Lemos, Liendo, Peelaers, Rastelli, Bolt van Rees]
  - **Schur operators** $[\mathcal{O}](z)$: cohomology of special $\mathcal{Q}$
  - Counted by **Schur index** $I_{\text{Schur}} \equiv q^{c_{4\text{d}}/2}\,\text{tr}(-1)^F q^{E-R}$:
  - $\mathcal{N} = (2,2)$ defects: non-vacuum modules [Cordova, Gaiotto, Shao]
  - Modular differential equations [Beem, Rastelli]
- Localization techniques Nekrasov, Pestun, Kapustin, Willet, Yaakov, Hama, Hosomichi, and many more

Questions

- Inspired by a 3d story [Dedushenko, Pufu, Yacoby]
- **Localization** in the context of this SCFT/VOA

- Inspired by a 3d story [Dedushenko, Pufu, Yacoby]
- **Localization** in the context of this SCFT/VOA

- With localization

Schur index = torus partition function

Correlation functions of Schur operators

Surface defects

- I_{Schur} as $S^3 \times S^1$ partition function
- Rigid supersymmetry [Closset, Dumitrescu, Festuccia, Seiberg, ...] backgrounds (a τ-family) and Killing spinors

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  - SYM action is $\mathcal{Q}$-exact: localizing term

The index

- I_{Schur} as $S^3 \times S^1$ partition function
- Rigid supersymmetry [Closset, Dumitrescu, Festuccia, Seiberg, ...] backgrounds (a τ-family) and Killing spinors
- On this geometry:
  - SYM action is $\mathcal{Q}$-exact: localizing term
- Standard localization: onto a T^2
  - BPS configs: flat connections $\mathfrak{a}$ and $Q, \tilde{Q}$ on $T^2 \subset S^3 \times S^1$
  - $S_{\text{HM}} \xrightarrow{\text{BPS}} S^{T^2}_{\beta\gamma}[Q, \tilde{Q}]$, $\quad \int D[\text{HM}] \to \int DQD\tilde{Q}$
  - $S_{\text{VM}} \xrightarrow{\text{fluctuation}} S^{T^2}_{bc}[b, c]$, $\quad \int D[\text{VM}] \to \int DbDc$

$$I_{\text{Schur}} = Z^{S^3 \times S^1} \xrightarrow{\text{Loc.}} \int d\mathfrak{a} DQD\tilde{Q}DbDce^{-S^{T^2}_{bc\beta\gamma}}$$

- Schur operator insertions?
  Naive Schur letters $Q, \tilde{Q}, D_z, \lambda_z \equiv (\lambda^I \sigma_z \tilde{\xi}_I), \ldots$

The correlation functions

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- "Generalize" the localization argument: **localizability**

$$\frac{d}{dt}\int D[\Phi]\mathcal{O}(z_i;t)\,e^{-S_0 - t\mathcal{Q}V} \sim \langle \partial_t \mathcal{O} - \mathcal{O}\mathcal{Q}V\rangle_t = 0$$

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- Serious Schur letters $Q, \tilde{Q}, D_z, \sqrt{t}\lambda_z, \sqrt{t}\tilde{\lambda}_z \Rightarrow \mathcal{O}(z;t)$

Schur operators are localizable

- Finally: a matrix integral formula for Schur correlation functions on $S^3 \times S^1$

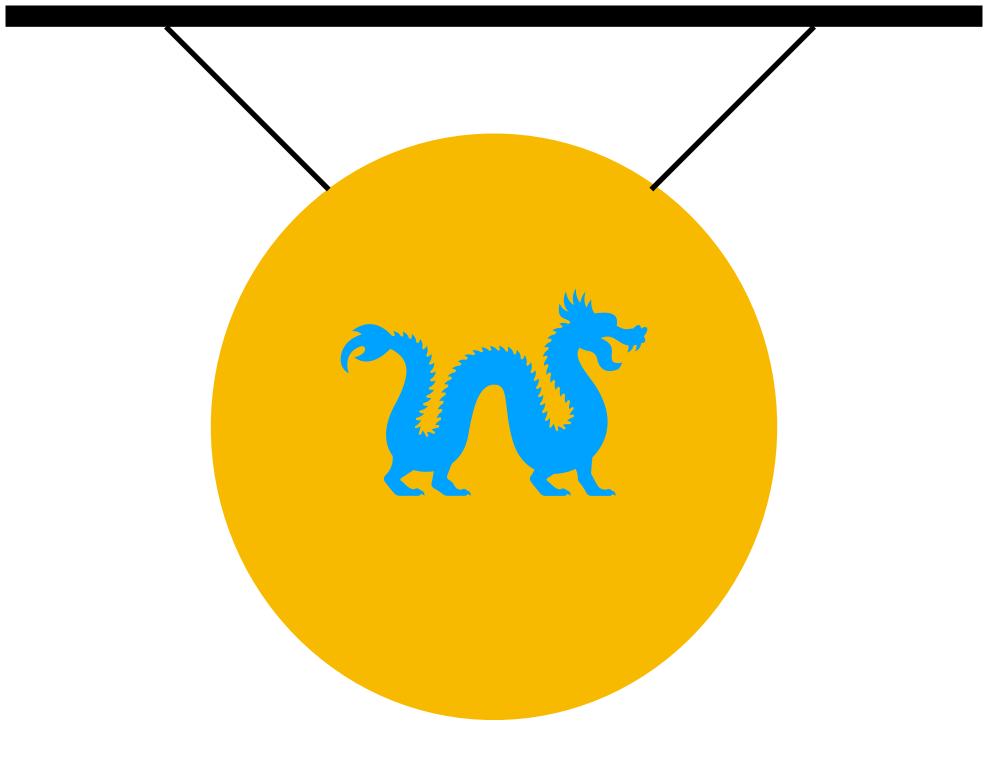
More

- I_{Schur} satisfies modular differential equations [Beem, Rastelli]

  Additional solutions could be accessed by including surface defects in localization

- Reduction along the time S^1: relate to deformation quantization story [Beem, Peelaers, Rastelli][Chester, Lee, Pufu, Yacoby]

Thank you!



Fin