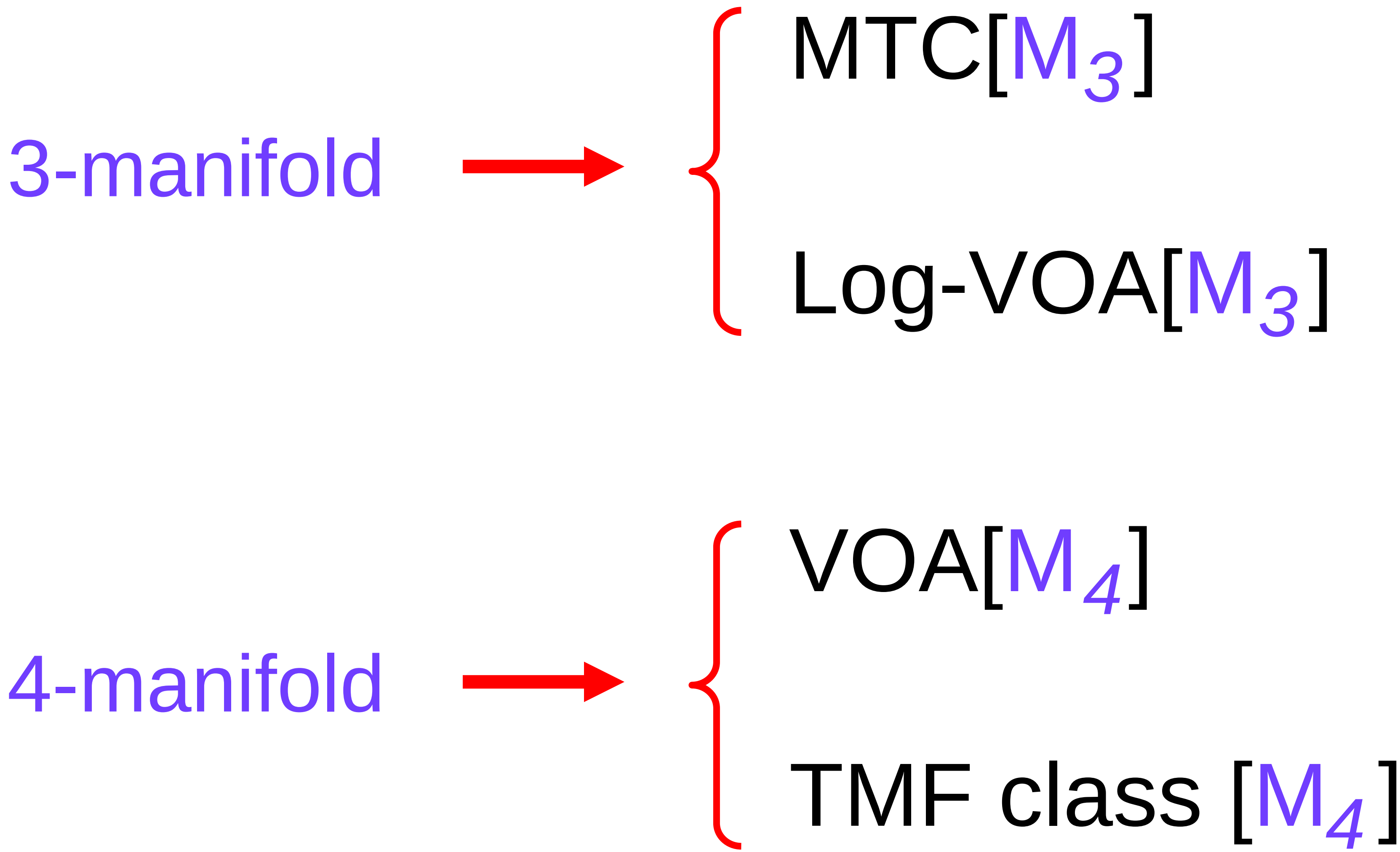

3-manifold
MTC[M_3]
Log-VOA[M_3]
4-manifold
VOA[M_4]
TMF class [M_4]

Surprise #1: M_4 = ALE space

Moduli problem:

$$F_A^+ + \ldots = 0$$

$$\sum_n q^n \, \chi \left(\mathcal{M}_n^{\text{inst}}(M_4) \right) \; = \; \chi_{\text{VOA}}(q)$$

[K.Yoshioka]
[H.Nakajima]
[C.Vafa, E.Witten]
:

Surprise #2:

[E.Witten]
[J.Bryan, R.Wentworth]
[A.Losev, N.Nekrasov, S.Shatashvili]
⋮
[M.Dedushenko, S.G., P.Putrov]

$$\begin{cases} F_A^+ = \sum_{i=1}^{N_f} \left(\Psi_i \overline{\Psi}_i \right)^+ \\ \not{D} \Psi_i = 0 \qquad i = 1, \ldots, N_f \end{cases}$$

$$\text{virtual dim } \mathcal{M}_{N_f}(M_4; \lambda) = \frac{N_f(\lambda^2 - \sigma) - 2(\chi + \sigma)}{4}$$

$\underline{N_f > 1}$: $\int_{\mathcal{M}_{N_f}} (\ldots)$ ill-defined

non-compact

Surprise #2:

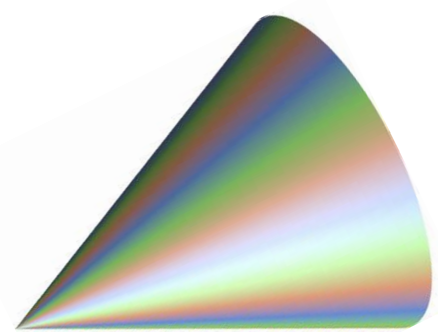
[E.Witten]
[J.Bryan, R.Wentworth]
[A.Losev, N.Nekrasov, S.Shatashvili]
:
[M.Dedushenko, S.G., P.Putrov]

$$\begin{cases} F_A^+ = \sum_{i=1}^{N_f} \left(\Psi_i \overline{\Psi}_i\right)^+ \\ \not{D}\Psi_i = 0 \qquad i = 1, \ldots, N_f \end{cases}$$

$$\text{virtual dim } \mathcal{M}_{N_f}(M_4; \lambda) = \frac{N_f(\lambda^2 - \sigma) - 2(\chi + \sigma)}{4}$$

Example ($N_f = 2\,,\ \lambda = 0$):

$$\Psi_1 = \text{harmonic spinor}$$
$$\Psi_2 = J\Psi_1$$



$$SU(N_f) \circlearrowright \mathcal{M}_{N_f}(M_4;\lambda)$$

<u>Theorem:</u> components of the fixed point set

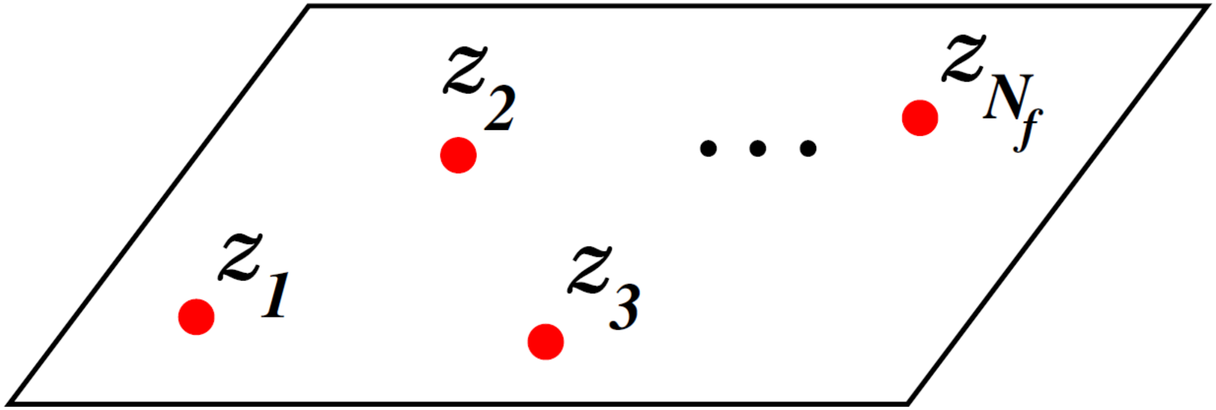
$$F_i \cong \mathcal{M}_1 \hookrightarrow \mathcal{M}_{N_f} \qquad i = 1, \ldots, N_f$$

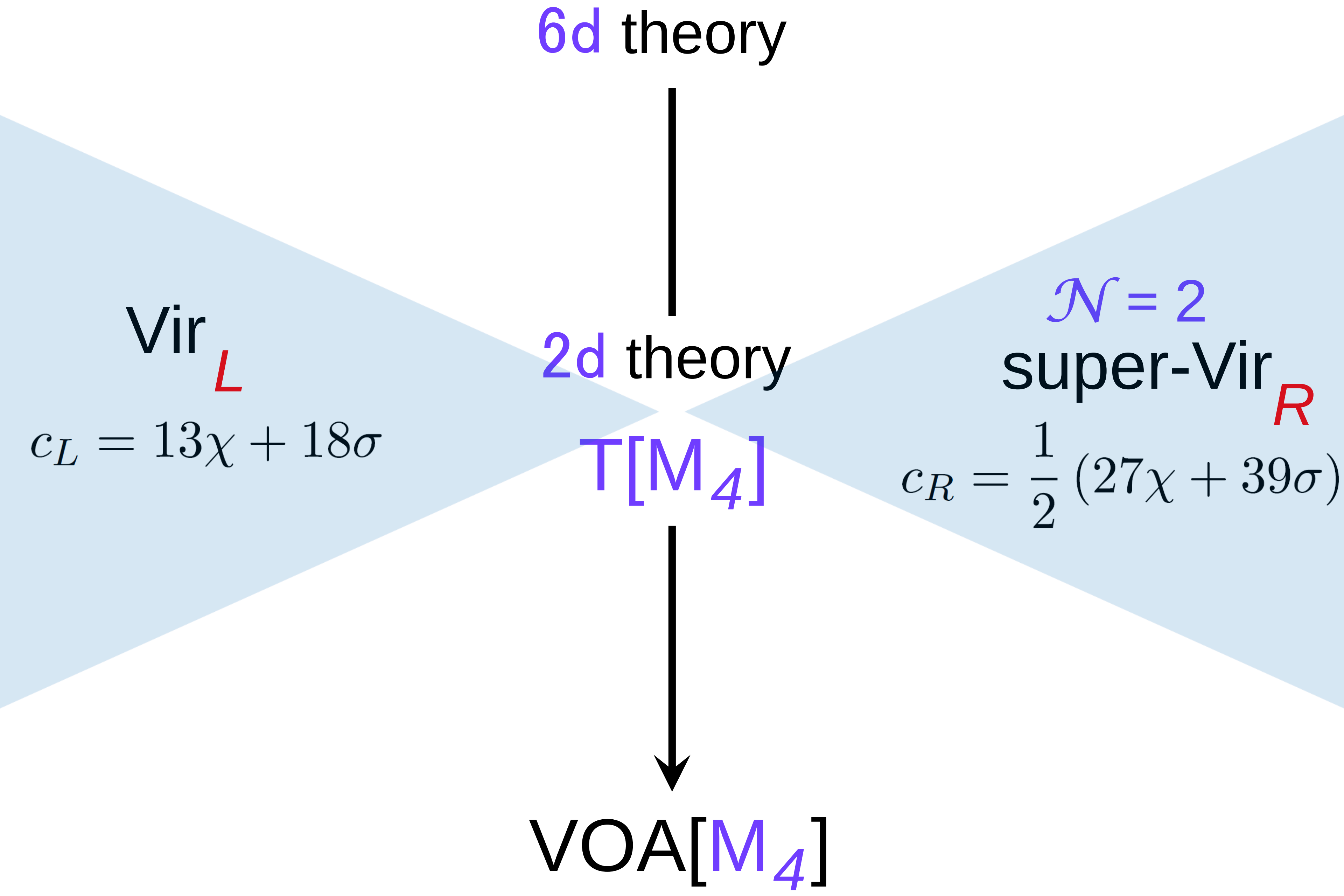
$$\dim_{\mathbb{C}} N_i = (N_f - 1)\mathrm{Ind}_{\mathbb{C}}(\not{D}) = \frac{N_f - 1}{8}(\lambda^2 - \sigma)$$

$$\mathrm{Eul}(N_i) = \prod_{j \neq i}(z_j - z_i)^{\frac{1}{8}(\lambda^2 - \sigma)}$$

$$\text{equivariant} \int_{\mathcal{M}_{N_f}} (\ldots) = \mathrm{SW}(\lambda) \sum_{i=1}^{N_f} \frac{1}{\prod_{j \neq i} (z_j - z_i)^{\frac{1}{8}(\lambda^2 - \sigma)}}$$

$$= \langle 0 | \, \mathcal{S}(z_1) \ldots \mathcal{S}(z_{N_f}) \, | \lambda \rangle_{\mathrm{VOA}[M_4]}$$



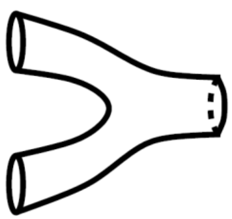


6d theory
2d theory
T[M4]
Vir L
$c_L = 13\chi + 18\sigma$
$\mathcal{N} = 2$
super-Vir R
$c_R = \frac{1}{2}(27\chi + 39\sigma)$
VOA[M4]

VOA[M_4] is known for many 4-manifolds

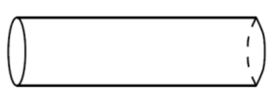
M_4	c_L	c_R
S^4	$26 = 2 + 24$	$27 = 3 + 24$
$\mathbb{C}\mathbf{P}^2$	57	60
$\mathbb{C}\mathbf{P}^1 \times \Sigma_{g,n}$	$2g + 4n + 4$	$6n + 6$

VOA [



] = $\beta\gamma$ system associated to $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$

VOA [



] = BRST reduction with respect to $\widehat{\mathfrak{sl}(2)}$

[P.Putrov, J.Song, W.Yan]

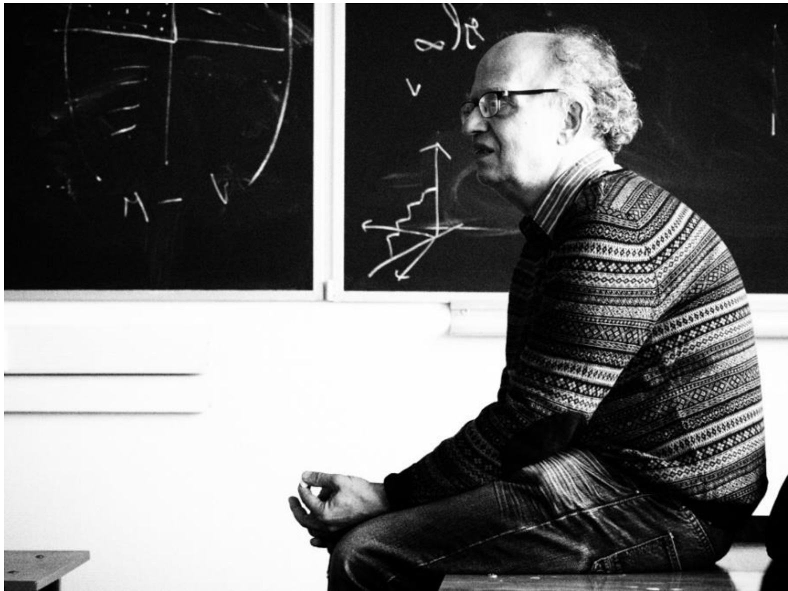
M_4	c_L	c_R
S^4	$26 = 2 + 24$	$27 = 3 + 24$
$\mathbb{C}\mathbf{P}^2$	57	60
$\mathbb{C}\mathbf{P}^1 \times \Sigma_{g,n}$	$2g + 4n + 4$	$6n + 6$
$m\mathbb{C}\mathbf{P}^2 \# n\overline{\mathbb{C}\mathbf{P}}^2$	$26 + 31m - 5n$	$27 + 33m - 6n$

Equivalences (e.g. trialities)

$\cong$

Kirby moves

[B.Feigin, S.G.]
[A.Gadde, S.G., P.Putrov]



$$\mathrm{VOA}\Big[\, M_4 \,\#\, \overline{\mathbb{C}\mathbf{P}}^2 \,\Big] \;\cong\; \mathcal{U} \otimes \mathrm{VOA}[M_4]$$

$$c_L(\mathcal{U}) \;=\; -5$$

$$\mathrm{VOA}\Big[\, M_4 \,\#\, \mathbb{C}\mathbf{P}^2 \,\Big] \;\cong\; \overline{\mathcal{U}} \otimes \mathrm{VOA}[M_4]$$

$$c_L(\overline{\mathcal{U}}) \;=\; +31$$

G	$SU(2)$	$SU(3)$	$SU(4)$	$SU(5)$	$\ldots$
$c_L(\mathcal{U})$	-5	-22	-57	-116	$\ldots$

$$\mathcal{T}_{N_1,N_2,N_3} \cong \mathcal{T}_{N_3,N_1,N_2} \cong \mathcal{T}_{N_2,N_3,N_1}$$

$$Gr(k,N) \cong Gr(N-k,N)$$

$$S \rightarrow Gr(k,N) \quad \cong \quad Q^* \rightarrow Gr(N-k,N)$$

$$Q \rightarrow Gr(k,N) \quad \cong \quad S^* \rightarrow Gr(N-k,N)$$

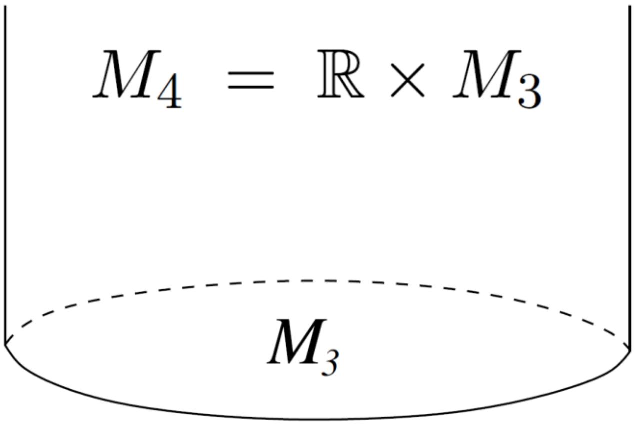
where

$$0 \longrightarrow S \longrightarrow \mathcal{O}^N \longrightarrow Q \longrightarrow 0$$

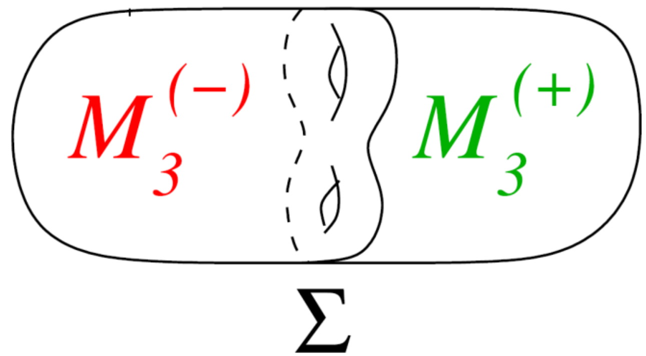
$$\mathcal{T}_{N_1,N_2,N_3} := \begin{array}{c} \Pi S^{\oplus N_3} \oplus \Pi Q^{\oplus N_2} \\ \downarrow \\ Gr\left(\frac{N_1+N_2-N_3}{2}, N_1\right) \end{array}$$


MTC
VOA
VO

Surprise #3: hidden SL(2,$\mathbb{Z}$) action on …

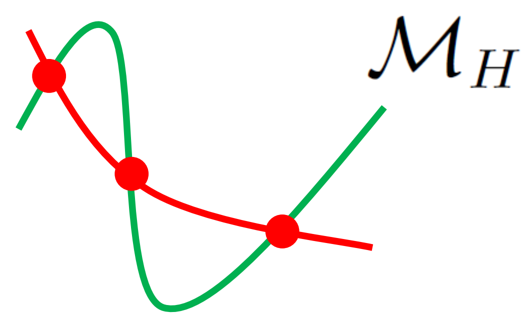


$\langle G_{\mathbb{C}}$ flat connections $\rangle$

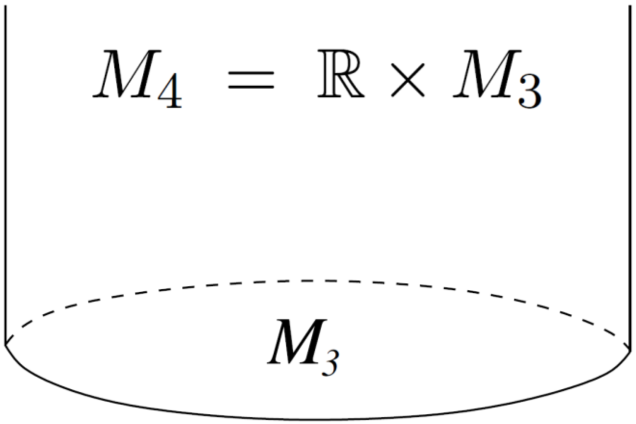


"Heegaard branes" in

$$\mathcal{M}_H(G,\Sigma) \cong \mathcal{M}_{\text{flat}}(G_{\mathbb{C}},\Sigma)$$



$SL(2,\mathbb{Z}) \circlearrowright \mathrm{HF}_{G_{\mathbb{C}}}(M_3)$



$$\overset{\otimes\, \mathbb{C}(q)}{\sim} \ \mathrm{Skein}(M_3)$$

$$= \mathrm{HP}(M_3) + \text{(reducibles)}$$

$$= K^0(\mathrm{MTC}[M_3, G])$$

[S.G., P.Putrov, C.Vafa]

Prediction:

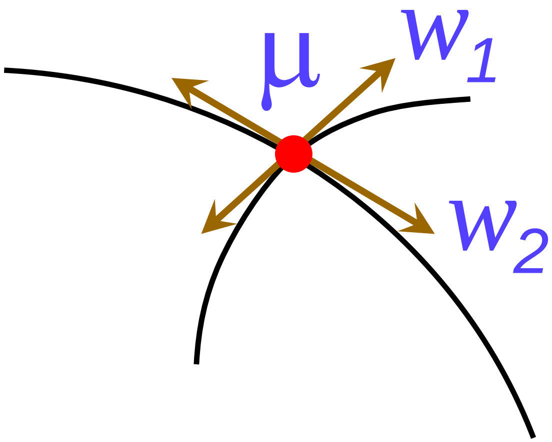
G	$SU(2)$	$SU(3)$	$SU(4)$	$SU(5)$	$SU(6)$	$\ldots$
$\dim_{\mathbb{Q}(q)} \mathrm{Sk}(T^3)$	9	29	74	129	261	$\ldots$

w/ S.Nawata, D.Pei, I.Saberi, ...

Surprise #4:

SL(2,$\mathbb{Z}$) $\circlearrowright$ $\left\langle \begin{array}{c}\text{components} \\ \text{of } U(1)_\beta \text{ fixed} \\ \text{point set}\end{array} \right\rangle$

Higgs bundles: $(A, \Phi) \mapsto (A, e^{i\theta}\Phi)$



Example:

wild

$$\mathcal{M}_H\Big($$  $$,\ \text{ramification}\Big)$$

2 fixed points

$\mu =$ 0 $1/5$

$w_1 =$ $2/5$ $6/5$

$w_2 =$ $3/5$ $-1/5$

Fibonacci MTC

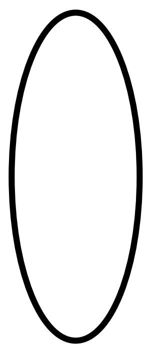
$$S = \frac{2}{\sqrt{5}} \begin{pmatrix} \sin\frac{\pi}{5} & \sin\frac{2\pi}{5} \\ \sin\frac{2\pi}{5} & -\sin\frac{\pi}{5} \end{pmatrix}$$

$$T = \begin{pmatrix} e^{-\frac{\pi i}{15}} & 0 \\ 0 & e^{\frac{11\pi i}{15}} \end{pmatrix}$$

[M.Dedushenko, S.G., H.Nakajima, D.Pei, K.Ye]

4d $\mathcal{N} = 2$ theory on $M_4 = S^1 \times M_3$

↳ 3d TQFT on M_3



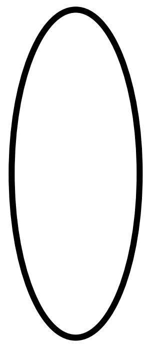
⇝ MTC (Coulomb branch)

cf. Turaev's theorem

[M.Dedushenko, S.G., H.Nakajima, D.Pei, K.Ye]

4d $\mathcal{N} = 2$ theory on $M_4 = S^1 \times M_3$

3d TQFT on M_3

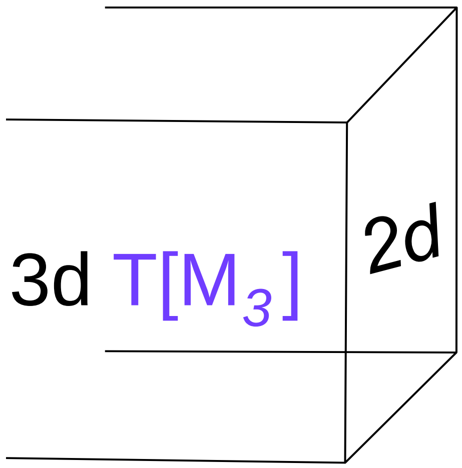
 $\rightsquigarrow$ MTC $\Big(S^1 \times$  $\Big)$

cf. Turaev's theorem

[M.Dedushenko, S.G., H.Nakajima, D.Pei, K.Ye]

Surprise #5:

$$\widehat{Z}_a(M_3) \;=\; q^{\Delta}\left(c_0 + c_1 q + c_2 q^2 + \ldots\right) \qquad c_i \in \mathbb{Z}$$



$$= \chi_{\text{log-VOA}}$$

see Miranda's talk

3-manifold $\xrightarrow{?}$ Log-VOA[M_3]

Surprise #6: 6d theory

w/ D.Pei, P.Putrov, C.Vafa

2d (0,1) theory

$\mathrm{T}[\mathrm{M}_4] \in \pi_d(\mathrm{TMF})$

graded by

$$d = 2(c_R - c_L)$$

Surprise #6: 6d theory


w/ D.Pei, P.Putrov, C.Vafa

2d (0,1) theory

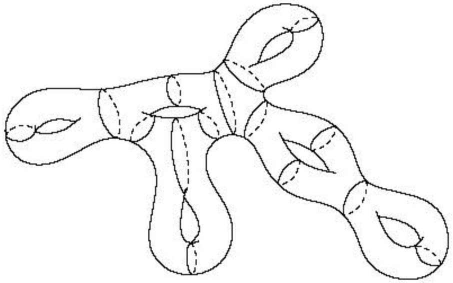
"equivariant TMF" ?

$$\mathrm{T}[M_4] \in \pi_* \mathrm{TMF}_G$$

graded by

$$\mathcal{A}_{\mathrm{2d}}^{G} := \mathrm{Hom}\left(\Omega_4^{\mathrm{Spin}}(BG), \mathbb{Z}\right) \oplus \mathrm{Hom}\left(\mathrm{Tor}\,\Omega_3^{\mathrm{Spin}}(BG), U(1)\right)$$

3d $\mathcal{N} = 2$



on 2-manifold → MTC

4d $\mathcal{N} = 2$ → MTC

5d $\mathcal{N} = 1$ → TMF

6d $\mathcal{N} = (0,2)$ → VOA


LEGO
CLASSIC
4-manifolds
Ages/edades
4-99
10693
LEGO® Creative Supplement
303 pcs/pzs
Building Toy
Jouet de Construction
Juguete para Construir
WARNING: CHOKING HAZARD. Small parts. Not for children under 3 years.
LEGO
3-manifolds
3 IN 1
7-12
31076