String perspectives on manifolds with G2 structure

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X. de la Ossa, ML, E. Svanes (1607.03473, 1704.08717, 1709.06974)
X. de la Ossa, ML, M. Magill, E. Svanes (1904.01027)

"To think freely is great,...

"To think freely is great, but to think rightly is greater"
(Thomas Thorild, 1759-1808)

Motivation and summary

Study compactifications of heterotic string on manifolds with G_2 structure.

Math motivation

New perspective on geometry, deformations, invariants,...

In particular: understand coupled moduli space of vector bundle and geometry.

Physics motivation

Determine effective field theory of heterotic compactifications.

Motivation and summary

Heterotic string to $\mathcal{O}(\alpha')$

Green, Schwarz:84, Gross *et.al.*:85, Bergshoeff, deRoo:89

- Bosonic fields: Metric G, B-field B, dilaton ϕ, gauge field A for $G \subset E_8 \times E_8$
- Fermionic fields: Gravitino, dilatino, gaugino

Compactifications $\mathcal{M}_{10} = \mathcal{M}_E \times Y$

- SUSY $\iff$ $\exists$ spinor λ on Y, nowhere vanishing, Killing: $\nabla_H \lambda = 0$

  $\exists$ connection A on $V \to X$, $\gamma^{mn} F_{mn}(A) = 0$

- Anomaly cancellation $H = \mathrm{d}B + \frac{\alpha'}{4}(CS(A) - CS(\Theta))$

- 7-manifold Y: $\lambda \iff G_2$ structure φ

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Motivation and summary

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Key ideas:

- Reformulate heterotic system as nilpotent differential $\check{\mathcal{D}}$

  $\check{\mathcal{D}}$ acts on $\mathcal{Q}$-valued forms, where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$.
- Infinitesimal moduli counted by $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$.
- Comparison with 4D $\mathcal{N} = 1$ Strominger–Hull system.

  cf. talk by Lara Anderson
- Superpotential.
- Conclusions and outlook.

G_2 structures

Bonan:66, Fernandez–Gray:82, Bryant:87,03, Hitchin:00, Joyce:00, Chiossi–Salamon:02

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$

- (Y,φ) has G_2 *structure* specified by non-degenerate associative 3-form φ

  Comment: true whenever Y is orientable and spin (and $\pi_1(Y)=0$).

- $\varphi \to$ Riemannian metric g_φ on Y, and a coassociative 4-form $\psi = *\varphi$

- Heterotic compactifications:

  SUSY constrains $\mathrm{d}\varphi$ and $\mathrm{d}\psi$, i.e. the torsion of the G_2 structure

G_2 structures

Fernandez–Ugarte:98, Friedrich–Ivanov:01, Gauntlett et.al.:01, ...

- (Y, φ) has G_2 *structure* specified by non-degenerate associative 3-form φ
- Torsion: decompose into *torsion classes* $\sim$ irreps of G_2:

$$\mathrm{d}\varphi = \tau_0\,\psi + 3\,\tau_1 \wedge \varphi + *\tau_3 \ , \qquad \Lambda^4 = \Lambda^4_1 \oplus \Lambda^4_7 \oplus \Lambda^4_{27} \ ,$$
$$\mathrm{d}\psi = 4\,\tau_1 \wedge \psi + *\tau_2 \ , \qquad \Lambda^5 = \Lambda^5_7 \oplus \Lambda^5_{14} \ .$$

- Heterotic compactifications:

SUSY $\iff$ τ_0 constant, $2\tau_1 = \mathrm{d}\phi$, $\boxed{\tau_2 = 0}$ $\rightsquigarrow$ integrable G_2 structure.

$\Lambda^k(Y)$ decomposes into $\Lambda^k_p(Y)$, p denotes G_2 irrep. Find these using φ:

Example: $\Lambda^1 = \Lambda^1_7 = T^*Y \cong TY$
$\implies$ *any* $\beta \in \Lambda^2$ decomposes as $\beta = \alpha \lrcorner \varphi + \gamma$, where $\alpha \in \Lambda^1$ and $\gamma \lrcorner \varphi = 0$

G_2 structures

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G_2 structures

- (Y, φ) has G_2 *structure* specified by non-degenerate associative 3-form φ
- Heterotic compactifications:

  SUSY $\Longrightarrow$ $\boxed{\tau_2 = 0}$ $\rightsquigarrow$ integrable G_2 structure:

  $$\boxed{\mathrm{d}\varphi = i_H(\varphi)\ ,\ \mathrm{d}\psi = i_H(\psi)} \text{ where } H = \tfrac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner_7\,\psi - \tau_3\ .$$

- No H-flux $\iff$ Y has G_2 holonomy
- H: torsion of unique G_2 compatible connection $\nabla\varphi = 0 = \nabla\psi$

Encode geometry by a differential? *cf.* Dolbeault differential on complex manifold

G_2 structures

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Encode geometry by a differential? *cf.* Dolbeault differential on complex manifold

G_2 structures: canonical G_2 cohomology

Decomposition of de Rham cohomology

Reyes-Carrion:93,Fernandez–Ugarte:98

Analogue of Dolbeault operator: project d onto G_2 irreps.

- Define differential operator $\check{\mathrm{d}}$ by

$$\check{\mathrm{d}}_0 = \mathrm{d}\ , \quad \check{\mathrm{d}}_1 = \pi_7 \circ \mathrm{d}\ , \quad \check{\mathrm{d}}_2 = \pi_1 \circ \mathrm{d}\ .$$

- Can show $\boxed{\tau_2 = 0 \iff \check{\mathrm{d}}^2 = 0}$ *cf.* Dolbeault differential $\bar{\partial}$ $\rightsquigarrow$ "integrable G_2 structure"

- Differential, elliptic complex

$$0 \to \Lambda^0(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^1(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^2_7(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^3_1(Y) \to 0$$

$\to$ canonical G_2-cohomology $\boxed{H^*_{\check{\mathrm{d}}}(Y)}$ *cf.* Dolbeault cohomology $H^*_{\bar{\partial}}(X)$

G_2 instanton bundle

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$

SUSY constrains $F(A)$: $\boxed{F(A)\wedge\psi=0} \iff A$ is G_2 instanton

Reyes-Carrion:93, 98,Fernandez–Ugarte:98

Canonical G_2-cohomology for instanton bundle

(*cf.* holomorphic bundles on complex manifolds)

- Recall: $\tau_2 = 0 \iff \check{\mathrm{d}}^2 = 0$

$$\check{\mathrm{d}}_0 = \mathrm{d}\ , \quad \check{\mathrm{d}}_1 = \pi_7 \circ \mathrm{d}\ , \quad \check{\mathrm{d}}_2 = \pi_1 \circ \mathrm{d}\ .$$

- Generalizes to vector bundles V with instanton connection:

$$\boxed{\tau_2 = 0 \text{ and } F(A)\wedge\psi = 0 \iff \check{\mathrm{d}}_A^2 = 0}$$

Anomaly cancellation condition

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$

Anomaly cancellation condition

$$\mathrm{d}B + \frac{\alpha'}{4}(CS(A) - CS(\Theta)) = H = \frac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner_7\,\psi - \tau_3\ .$$

or, Bianchi idendity

$$\frac{\alpha'}{4}(\mathrm{tr}F(A)\wedge F(A) - \mathrm{tr}R(\Theta)\wedge R(\Theta)) = \mathrm{d}H = \mathrm{d}\left(\frac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner_7\,\psi - \tau_3\right)\ .$$

Other conditions?

Hull:86, Ivanov:10, Martelli–Sparks:10

SUSY+anomaly cancellation $\implies$ EOM $\iff$ Θ is a G_2 instanton

$$R(\Theta)\wedge\psi = 0$$

Geometry as differential

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$:

1. (Y, φ) 7-manifold with integrable G_2 structure
2. $(V, A), (TY, \Theta)$ G_2 instanton bundles
3. Anomaly cancellation $\mathrm{d}B + \frac{\alpha'}{4}(CS(A) - CS(\Theta)) = H = \frac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner_7\,\psi - \tau_3$

Encode constraints as nilpotency $\check{\mathcal{D}}^2 = 0$ of suitable differential $\check{\mathcal{D}}$

Geometry as differential

Want: Heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H] \iff \check{\mathcal{D}}^2 = 0$

- Start with

$$\mathcal{D} = \begin{pmatrix} \mathrm{d}_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & \mathrm{d}_\Theta & 0 \\ \mathcal{F} & 0 & \mathrm{d}_A \end{pmatrix}$$

- $\mathcal{D}$ acts on $\mathcal{Q}$-valued forms where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$
- $\mathrm{d}_A = \mathrm{d} + A\wedge$ etc.
- d_ζ has torsion $-H$ (*cf.* G_2 connection ∇_H)
- $\mathcal{F}$, $\mathcal{R}$ linear maps

$$\mathcal{F}: \quad \Omega^p(Y, T^*Y) \oplus \Omega^p(Y, \mathrm{End}(V)) \longrightarrow \Omega^{p+1}(Y, \mathrm{End}(V)) \oplus \Omega^{p+1}(Y, T^*Y)$$

$$\mathcal{F}(M) = (-1)^p\, g^{ab}\, M_a \wedge F_{bc}\, \mathrm{d}x^c \ ,$$

$$\mathcal{F}(\alpha)_a = (-1)^p\ \frac{\alpha'}{4}\, \mathrm{tr}(\alpha \wedge F_{ab}\, \mathrm{d}x^b) \ .$$

Geometry as differential

Want: Heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H] \iff \check{\mathcal{D}}^2 = 0$

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- $\mathcal{D}$ acts on $\mathcal{Q}$-valued forms where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$
- Next, project to get $\check{\mathcal{D}}$

$$\check{\mathcal{D}}_0 = \mathcal{D} \ , \quad \check{\mathcal{D}}_1 = \pi_7 \circ \mathcal{D} \ , \quad \check{\mathcal{D}}_2 = \pi_1 \circ \mathcal{D} \ .$$

- The result follows.

Infinitesimal moduli

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$

Key ideas:

- Reformulate heterotic system as nilpotent differential $\check{\mathcal{D}}$ ✓
- Infinitesimal moduli counted by $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$

Naive infinitesimal moduli

Y: integrable G_2 structure manifold V: instanton gauge bundle

- $\delta_t\psi$ (determines $\delta_t\varphi$): geometric moduli: $H^3(Y)$ if $H=0$
  $H=0$: *Joyce:96, Dai, Wang, Wei:03, de Boer, Naqvi, Shomer:05,...*
- $\delta_t A$: Vector bundle moduli: $H^1(Y,\mathrm{End}(V))$
  $H=0$: *Sa-Earp:09,...*
- $\delta_t B$: deformations of B-field, $H = \mathrm{d}B + \alpha'(...)$

Infinitesimal moduli

Geometric moduli and canonical G_2 cohomology

- Let $\delta_t \psi = i_M(\psi)$, where $M_t \in \Lambda^1(Y, T^*Y)$.
- Preserve $\tau_2 = 0$: $\boxed{i_{\check{\mathrm{d}}_\zeta M_t}(\psi) = 0}$
- Diffeomorphisms: $\boxed{\mathcal{L}_V \psi = i_{\check{\mathrm{d}}_\zeta V}(\psi)}$ where $V \in \Lambda^0(Y, T^*Y)$

$$\implies \mathcal{TM}_Y = \{M_t : i_{\check{\mathrm{d}}_\zeta M_t}(\psi) = 0\} / \{M_t : M_t = \check{\mathrm{d}}_\zeta V\}$$

- But $\check{\mathrm{d}}_\zeta^2 \neq 0$: ζ is not an instanton

  Dimension of infinitesimal geometric moduli space is not finite, in general.
- Exception: G_2 holonomy $\quad \mathcal{TM}_Y \cong H^3_{\mathrm{d}}(Y) \cong H^1_{\check{\mathrm{d}}_\zeta}(Y, TY)$

Infinitesimal moduli

Deformations of $[(Y,\varphi),(V,A)]$

Want deformations that preserve $F \wedge \psi = 0$

$$\implies \check{\mathrm{d}}_A \alpha = \mathrm{d}_A \alpha_t \wedge \psi = -F \wedge i_{M_t}(\psi) = \check{\mathcal{F}}(M_t)$$

Bianchi identity $\mathrm{d}_A F = 0 \implies \check{\mathcal{F}}(\check{\mathrm{d}}_\zeta(M)) + \check{\mathrm{d}}_A(\check{\mathcal{F}}(M)) = 0$

- Vector bundle moduli $\mathrm{d}_A \alpha_t = 0 \rightsquigarrow H^1(Y, \mathrm{End}(V))$
- Geometric moduli must lie in kernel of map $\check{\mathcal{F}}$

  $$\implies \mathcal{TM}_{[(Y,\varphi),(V,A)]} = H^1(Y, \mathrm{End}(V)) \oplus \ker\check{\mathcal{F}} \ , \ \ker\check{\mathcal{F}} \subset \mathcal{TM}_Y$$
- Not enough to prove finiteness when $H \neq 0$

Infinitesimal moduli

Infinitesimal moduli and canonical G_2 cohomology

Consider, for $\mathcal{Z}_t = (M_t, \kappa_t, \alpha_t)^T \in \Lambda^1(Y, \mathcal{Q})$

$$\mathcal{D}\mathcal{Z}_t = \begin{pmatrix} \mathrm{d}_\zeta\, M_t + \mathcal{R}(\kappa_t) - \mathcal{F}(\alpha_t) \\ \mathrm{d}_\Theta \kappa_t + \mathcal{R}(M_t) \\ \mathrm{d}_A \alpha_t + \mathcal{F}(M_t) \end{pmatrix} \qquad \text{with}\, \mathcal{D} = \begin{pmatrix} \mathrm{d}_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & \mathrm{d}_\Theta & 0 \\ \mathcal{F} & 0 & \mathrm{d}_A \end{pmatrix}$$

Constraints on moduli* $\implies$ $\boxed{\check{\mathcal{D}}\mathcal{Z}_t = 0}$

*Subtlety: B-field variations do not decouple! $\rightsquigarrow$ antisymmetric part of matrix M_t

Diffeomorphisms and gauge symmetry $\implies$ $\boxed{\mathcal{Z}_{triv} = \check{\mathcal{D}}\mathcal{V}}$

$$\implies \quad \boxed{\mathcal{TM}_{[(Y,\varphi),(V,A),(TY,\Theta),H]} \cong H^1_{\check{\mathcal{D}}}(\mathcal{Q})} \qquad \text{(finite dimensional)}$$

Infinitesimal moduli

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  $\check{\mathcal{D}}$ acts on $\mathcal{Q}$-valued forms where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$
- Infinitesimal moduli $\sim$ $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$ ✓

Comparison of 3D and 4D heterotic $\mathcal{N} = 1$ systems

6D Strominger–Hull system $[(X, \Omega, \omega), (V, A), (TY, \Theta), H]$

- Gaugino $\rightarrow$ F-term/D-term
- Nilpotent differential $\bar{D}$

$$T^*_{(1,0)}X \oplus \mathrm{End}(TX) \oplus \mathrm{End}(V) \oplus T_{(1,0)}X$$

- $\bar{D}$ upper triangular
- Hol. Courant algebroid $\rightsquigarrow$ Hitchin's generalised geometry

7D Heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

- Gaugino $\rightarrow$ F-term
- Nilpotent differential $\check{\mathcal{D}}$

$$T^*Y \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V)$$

- $\check{\mathcal{D}}$ not upper triangular
- ——

  but, see Clarke *et.al.*:16

6D deformation theory *Atiyah:57, Kodaira, Spencer:58,60, Candelas, de la Ossa:91, Becker, Tseng, Yau:06, Anderson,et.al:10,11,13, Fu, Yau:11, Baraglia, Hekmati:13, Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez,et.al:13,15,...*

Superpotential

Alternative perspective:

Moduli space as critical locus of a superpotential on off-shell parameter space

Strategy

- Dimensional reduction $\implies$ 3D gravitino mass $\boxed{M_{3/2} = e^{K} W}$
  Remark: need Hessian K
- Check $\delta W = 0 \quad \iff \mathcal{N} = 1$ heterotic G_2 system
- Follows that $\delta^2 W \quad \iff$ equation for moduli

Superpotential: Dimensional reduction

Fermionic part of 10D heterotic supergravity action

Bergshoeff, de Roo:89, Gurrieri, Lukas, Micu:07

$\rightarrow$ 3D kinetic and mass terms for gravitino

$$S_{0,f} = -\frac{1}{2\kappa_{10}^2}\int_{M^{10}} d^{10}x\sqrt{-g}\, e^{-2\phi}$$
$$\left(\overline{\Psi}_M \Gamma^{MNP} D_N \Psi_P - \frac{1}{24}\left(\overline{\Psi}_M \Gamma^{MNPQR}\Psi_R + 6\,\overline{\Psi}^N \Gamma^P \Psi^Q\right) H_{NPQ}\right)$$

$$S_{3D} \supset -\frac{1}{2\kappa_3^2}\int d^3x\sqrt{-g}\,\left(\overline{\psi}_\mu \Gamma^{\mu\nu\kappa} D_\nu \psi_\kappa + m\overline{\psi}_\mu \Gamma^{\mu\kappa}\psi_\kappa\right)$$

Straightforward, but

- field normalisation (correct EH term)
- conventions for gravitino mass in AdS (want SUSY $\sim 0 = W = e^{-K/2} M_{3/2}$).

Superpotential: Dimensional reduction

To match 3D EH term decompose

$$
\begin{aligned}
g_{10} &= e^n\, g_3 \oplus g_7 \\
\Gamma^\mu_{(10)} &= e^{-n/2}\, \Gamma^\mu_{(3)} \otimes \mathrm{Id} \otimes \sigma_2 \\
\Gamma^i_{(10)} &= \mathrm{Id} \otimes \Gamma^i_{(7)} \otimes \sigma_1 \\
\Psi_\mu &= e^{n/2}(\rho_\mu \otimes \lambda \otimes \theta) \\
\bar{\Psi}_\mu &= (\bar{\rho}_\mu \otimes \lambda^\dagger \otimes \theta^\dagger \sigma_2)\,,
\end{aligned}
$$

Then, e.g.

$$
\begin{aligned}
&\int d^{10}X\sqrt{-g_{10}}\, e^{-2\phi}\left(-\frac{1}{24}\overline{\Psi}_\mu \Gamma^{\nu\mu}\Gamma^{ijk}\Psi_\nu H_{ijk}\right) \\
&= \int d^3x\sqrt{-g_3}\,(\bar{\rho}_\mu \Gamma^{\mu\nu}\rho_\nu)\left[\frac{1}{24}\int d^7y\sqrt{g_7}\, e^{-2\phi+n}(-i\lambda^\dagger \Gamma^{ijk}\lambda)H_{ijk}\right] \\
\implies\ & m \sim \frac{1}{4}\int *_7 H \wedge \varphi \cdot e^{-2\phi+n}\,.
\end{aligned}
$$

Superpotential: Dimensional reduction

Result:

Three mass contributions

$$\tilde{M}_{3/2} = -\frac{1}{8}\int_7 e^{-2\phi+n}\left(d\varphi\wedge\varphi - 2\,*_7\,H\wedge\varphi + 2\,*_7\,f\right)\,, \tag{1}$$

Fixing conventions so that SUSY $\implies$ $M_{3/2}=0$ (with $h=-\frac{2}{7}f$)

$$M_{3/2} = \frac{1}{4}\int_7 e^{-2\phi+n}\left(-\frac{1}{2}d\varphi\wedge\varphi + (H+h\varphi)\wedge\psi\right)\,.$$

Analysing the Einstein–Hilbert term, identify $K \simeq n \implies$

$$W = \frac{1}{4}\int_Y e^{-2\phi}\,\left((H+h\,\varphi)\wedge\psi - \frac{1}{2}\,\mathrm{d}\varphi\wedge\varphi\right)$$

Superpotential: Variations and SUSY

Want to show $\delta W = 0 \iff \mathcal{N} = 1$ heterotic G_2 system

$$\delta W = \int_Y e^{-2\phi} \Big\{ -2\,\delta\phi\Big((H + h\,\varphi)\wedge\psi - \frac{1}{2}\,\mathrm{d}\varphi\wedge\varphi\Big) - \mathcal{B}\wedge e^{2\phi}\mathrm{d}(e^{-2\phi}\,\psi) + \frac{\alpha'}{2}\,[\mathrm{tr}(\delta A\,F\wedge\psi) - \mathrm{tr}(\delta\Theta\,R(\Theta)\wedge\psi)] + +(H + h\varphi)\wedge\delta\psi + \delta\varphi\wedge\Big(h\,\psi - \mathrm{d}\varphi + \mathrm{d}\phi\wedge\varphi\Big)\Big\}$$

with $\delta H = \mathrm{d}\mathcal{B} + \frac{\alpha'}{2}(\mathrm{tr}(F\delta A) - \mathrm{tr}(R(\Theta)\,\delta\Theta))$.

Result: critical points of $W \iff$ heterotic G_2 system with $\tau_1 = \frac{1}{2}\,\mathrm{d}\phi$, $h = \frac{1}{3}\,\tau_0$, and $W = 0$ ✓

Conclusions and outlook

Conclusions

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Key ideas:

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  $\check{\mathcal{D}}$ acts on $\mathcal{Q}$-valued forms where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$
- Infinitesimal moduli $\sim$ $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$
- Superpotential W s.t. $\delta W = W = 0 \iff \mathcal{N} = 1$ heterotic G_2 system

Conclusions and outlook

Outlook

- Examples. Compute cohomologies?
  Fernandez, Ivanov, Ugarte, Villacampa:11, Walpuski:13, Menet, Nordström, Sá-Earp:15,...
- New perspective on Donaldson–Segal invariants?
  $W[\Phi] = W[\Phi_0] + \frac{1}{2}\int_M e^{-2\phi_0}\langle \mathcal{X}, \mathcal{D}\mathcal{X}\rangle \wedge \psi_0 + \mathcal{O}(\mathcal{X}^3)$
- Structure of moduli space: metric, Hessian potential, singularities?
- Higher order deformations: obstructions, integrability, Maurer–Cartan eq?
- Decompactification and relation to $SU(3)$ structure moduli spaces?

Conclusions and outlook

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Tack!