5d SCFTs, Flavors and BPS States

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String-Math 2019, Uppsala, July 2, 2019

1906.11820 and (to appear soon)^2, with

Fabio Apruzzi, Craig Lawrie, Ling Lin, Yi-Nan Wang
(Oxford/UPenn)

5d $\mathcal{N} = 1$ Gauge Theories and SCFTs

5d $\mathcal{N} = 1$ SCFTs are intrinsically non-perturbative quantum field theories. At low energies: there can be an effective descriptions in terms of weakly coupled gauge theories.

5d $\mathcal{N} = 1$ SUSY massless multiplets:

$$\begin{aligned} \text{Vector multiplet:} \quad & \mathcal{A} = (A_\mu, \phi^i, \lambda) \\ \text{Hyper-multiplet:} \quad & \mathbf{h} = (h \oplus h^c, \psi) \end{aligned}$$

Coulomb branch: $\langle \phi^i \rangle \neq 0$.

Example: [Seiberg][Morrison, Seiberg]

$G_{\text{gauge}} = SU(2)$ with N_F. Coulomb branch is 1d ("rank 1") and weakly couple flavor symmetry $SO(2N_F)$ enhances to E_{N_F+1} in the UV.

In general: difficult to come by properties of the UV fixed points.

Motivation

- Classification of 6d $(1,0)$ SCFTs: F-theory on elliptically fibered Calabi-Yau threefolds (CY3). [Heckman, Morrison, (Rudelius), Vafa][Bhardwaj]
- 6d to 5d:

  $\Rightarrow$ Compactification on S^1: 5d marginal theory (UV completes in 6d).

  $\Rightarrow$ Add masses to flavors: flows to a 5d UV fixed point.
- Can we identify and characterize all such 5d SCFTs?

  $\Rightarrow$ Yes and we propose a systematic, combinatorial way to do so
- Whether these are *all* 5d SCFTs remains to be seen (hinges on the classification of canonical singularities in CY3, and whether these are all obtained from elliptic models). See [Xie, Yau]

Proposal

[Apruzzi, Lawrie, Lin, SSN, Wang]

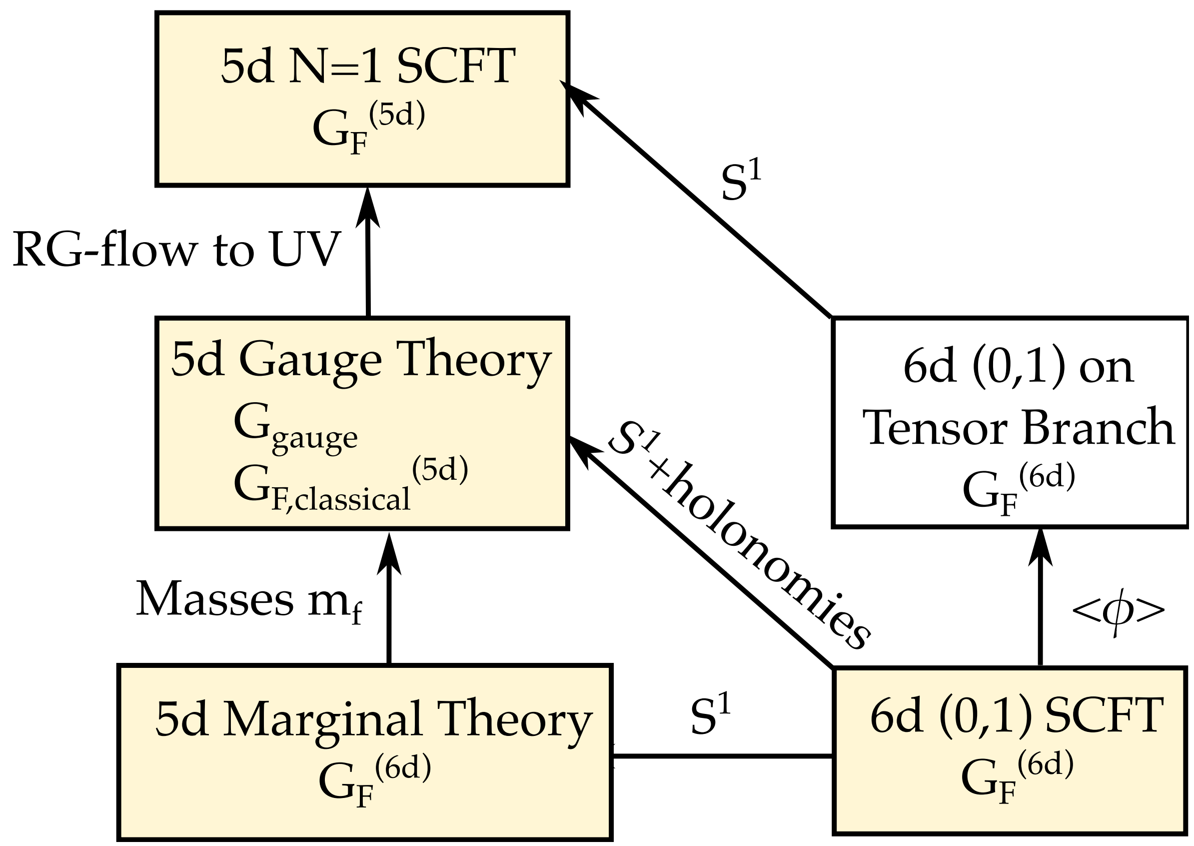
5d SCFTs from M-theory on elliptic CY3 with non-flat fibers

Extended Coulomb branch phases ($\langle\phi\rangle$ + Coulomb branch parameters to weakly gauge the classical flavors) are matched with geometric moduli space (resolutions of singular elliptic fibrations)

We find a systematic, combinatorial description in terms of graphs ("Combined Fiber Diagrams" (CFDs)) which describe 5d SCFTs including

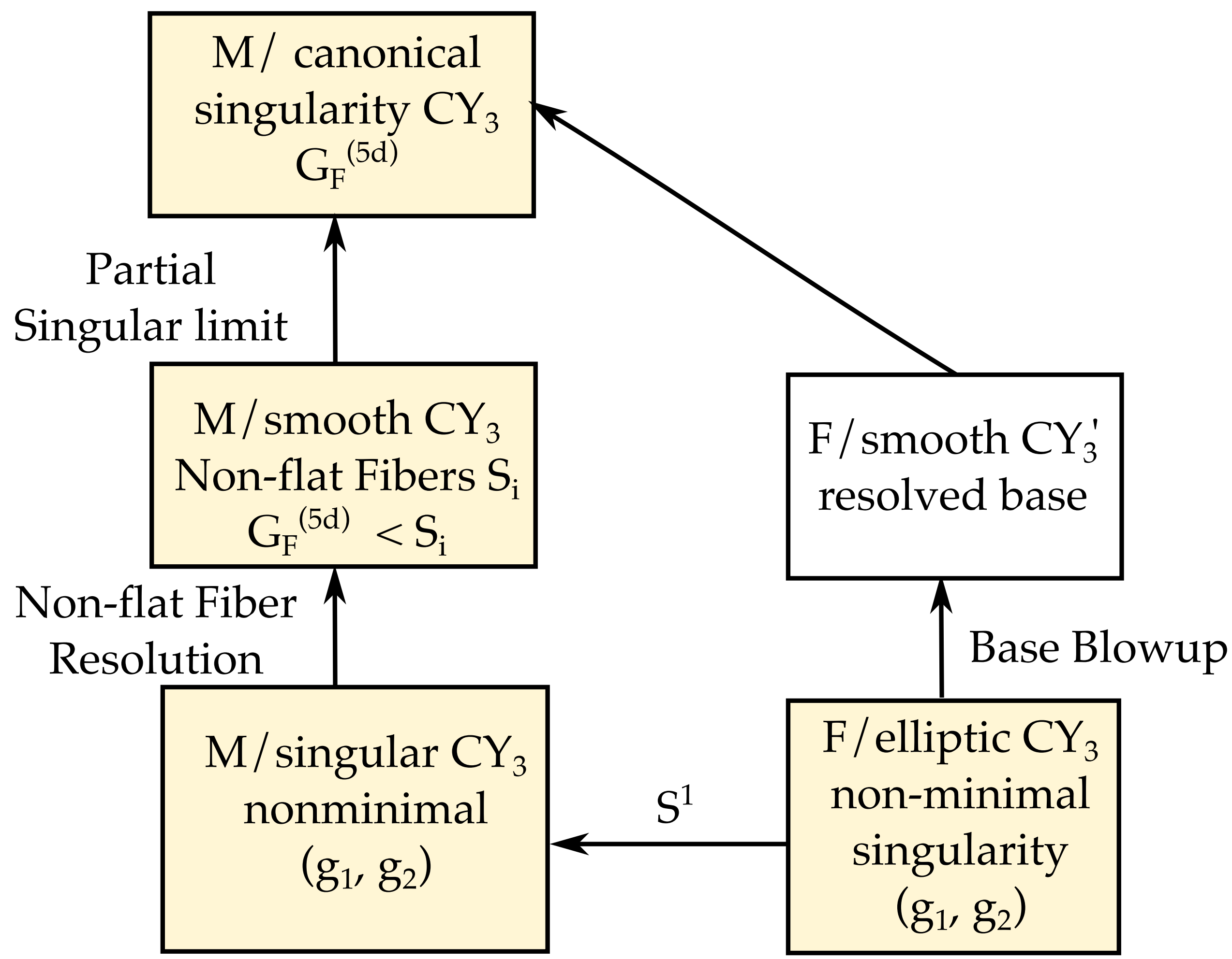
- $\star$ strongly coupled (usually enhanced) flavor symmetry
- $\star$ Mass deformations, i.e. descendant SCFTs
- $\star$ BPS states

General Strategy: 6d to 5d



Refined Strategy

[Apruzzi, Lawrie, Lin, SSN, Wang]



6d to 5d SCFT

Earlier developments: [Intrilligator, Morrison, Seiberg][Klemm, Mayr, Vafa][Aharony, Hanany, Kol]

Recently, different approach from 6d SCFT on S^1: [Del Zotto, Heckman, Morrison][Jefferson, Katz, Kim, Vafa][Bhardwaj, Jefferson][Apruzzi, Lin, Mayrhofer][Closset, Del Zotto, Saxena]. The strategy so far:

- Tensor branch of 6d on S^1 (base blowup)
- To reach SCFT usually need to flop surfaces in the CY3 $\Rightarrow$ does not retain elliptic fibration structure
- SCFT enhanced flavor symmetries not manifest in geometry

Main new idea [ALLSW]: non-flat resolution of the elliptic fibration, retains info about flavor symmetry, and even BPS states! Succinct description in terms of simple graphs "CFDs"

6d SCFTs from F-theory on CY3

F-theory on non-compact CY3:

- Elliptic CY3 $\mathbb{E}_\tau \hookrightarrow Y_3 \to B$, with a section has Weierstrass form

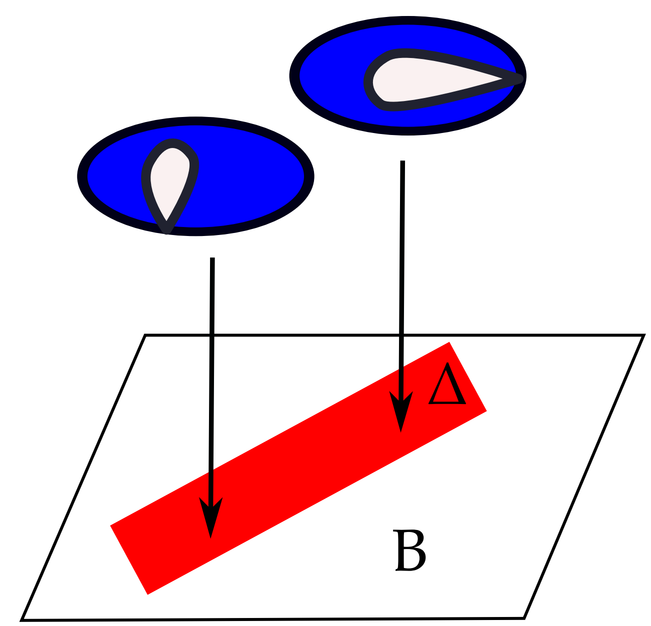
$$y^2 = x^3 + fx + g\,, \qquad f, g \in H^0(K_B^{-4/6})\,.$$

Noncompact base B: $\mathbb{C}^2$, coordinates u, v

- Discriminant: Singular fiber above $u = 0$:

$$\Delta = 4f^3 + 27g^2 = O(u^n)$$

- Kodaira fiber above $u = 0$, i.e. in codim 1.



Classification of Singular Fibers

Codim 1 in base: Kodaira classified singular fibers

$\boxed{\text{Singular fibers}} \quad \longleftrightarrow \quad \boxed{\text{(Decorated) ADE affine Dynkin diagram}}$

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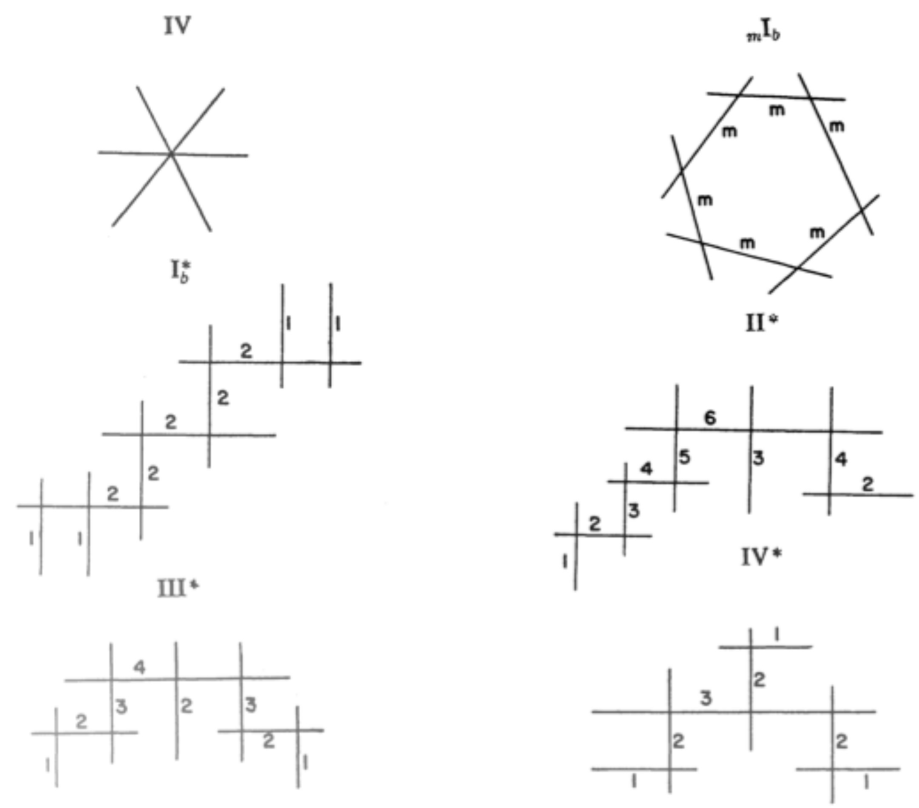


FIGURE 1. Each line represents $\Theta_{\rho s}$; the integer attached to the line gives $n_{\rho s}$.

	$\mathrm{ord}_S(f)$	$\mathrm{ord}_S(g)$	$\mathrm{ord}_S(\Delta)$	singularity	local gauge group factor
I_0	≥ 0	≥ 0	0	none	–
I_1	0	0	1	none	–
I_2	0	0	2	A_1	$SU(2)$
$I_m, m \geq 1$	0	0	m	A_{m-1}	$Sp([\frac{m}{2}])$ or $SU(m)$
II	≥ 1	1	2	none	–
III	1	≥ 2	3	A_1	$SU(2)$
IV	≥ 2	2	4	A_2	$Sp(1)$ or $SU(3)$
I_0^*	≥ 2	≥ 3	6	D_4	G_2 or $SO(7)$ or $SO(8)$
$I_m^*, m \geq 1$	2	3	$m+6$	D_{m+4}	$SO(2m+7)$ or $SO(2m+8)$
IV^*	≥ 3	4	8	E_6	F_4 or E_6
III^*	3	≥ 5	9	E_7	E_7
II^*	≥ 4	5	10	E_8	E_8
non-minimal	≥ 4	≥ 6	≥ 12	non-canonical	–

Kodaira's classification of singular fibers and gauge groups

Minimal versus non-minimal: codimension 2

In codimension 2: $u = v = 0$ in the base:

1. Minimal: ordinary bifundamental matter, and codimension two fiber are (monodromy-reduced) Kodaira fibers (collection of rational curves).

2. Non-mininal: Weierstrass model

$$y^2 = x^3 + f x + g \, , \qquad \text{ord}_{u=v=0}(f, g, \Delta) \geq (4, 6, 12)$$

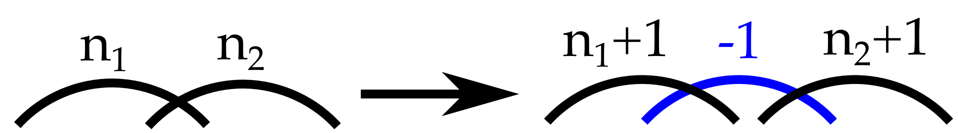
   no crepant resolution of fiber possible that keeps complex 1d fiber.
   Two options:

   (1) Blowup the base (6d approach)

   (2) Blowup fiber with non-flat fibers

Non-minimal Singularities: (1) Base-blowup

Blowup the base: $u = v = 0$ non-minimal point p: blowup by bluing in $\mathbb{P}^1$.
$\Sigma_i \cong \mathbb{P}^1$ with $\Sigma_i^2 = n_i$ (degree of the normal bundle in B)



Example: Conformal Matter

[del Zotto, Heckman, Tomasiello, Vafa]

Start with singularities $G_1 - G_2$ colliding at a point, e.g.

$$(E_6, E_6): \qquad y^2 = x^3 + u^4 v^4$$

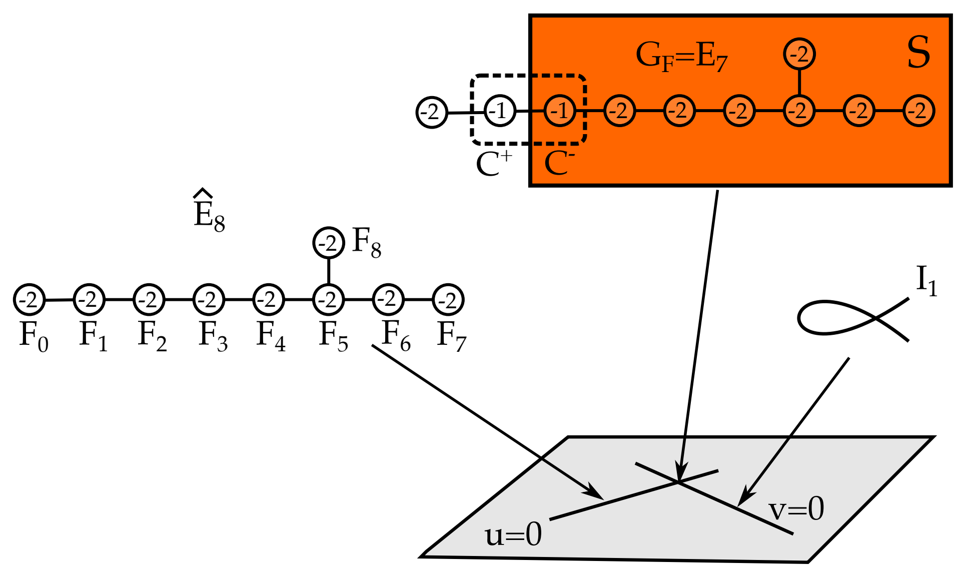
Blowup the point $u = v = 0$ repeatedly, until model is minimal:

$$[E_6](-1)(-3)(-1)[E_6]$$

Has E_6^2 flavor symmetry and an $\mathfrak{su}(3)$ gauge group on the -3 curve.

Non-minimal Singularities: (2) Non-Flat Fibers

Non-flat fiber resolution: crepant resolution, but includes complex surface components S_i, in addition to rational curves:



Physics:

Non-flat fiber surfaces are compact $S_i \leftrightarrow U(1)$ gauge fields.

Flavor $\mathbb{P}^1$s contained in S_i remain flavor symmetries as $\text{vol}(S_i) \to 0$ limit.

M/CY3 — 5d Dictionary

[Intriligator, Morrison, Seiberg]

New insight: for non-flat fibration: S_i are non-flat fiber components

1. S_i compact divisors $i = 1, \cdots, r = \text{rank} \Rightarrow \; U(1)^r$ gauge bosons
2. $S_i \to C_i$ (collapse to curve) $\Rightarrow$ enhancement of $U(1)^r \subset G_{\text{gauge}}$
3. $S_i \to$ point (collapse to point) $\Rightarrow$ strong coupling
4. Coulomb branch including hypers in $\mathbf{R}_F$ of $G^{5d}_{\text{F, classical}}$:

$$\mathcal{F}_{1-\text{loop}} = \frac{1}{12}\Big(\sum_{\alpha} |\phi \cdot \alpha|^3 - \sum_{\lambda_F \in \mathbf{R}_F} |\lambda_F \cdot \phi + m_F|^3\Big)$$

   $\Rightarrow$ Extended Kähler cone: m_F = mass deformations, or CB parameters for weakly gauging flavor symmetry.

5. Flavor symmetry: determined by ADE singularities over non-compact curves, whose fibral curves are contained in S_i
6. M2-wrapping modes on curves: BPS states

Example: The E-string and 5d rank 1 SCFTs

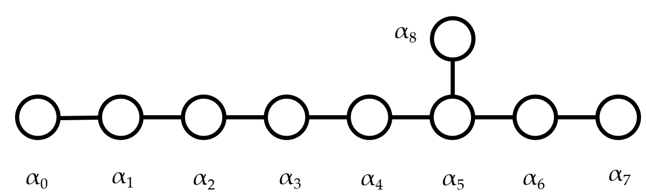
1. Starting point 6d: non-compact base with coordinates u, v: $E_8 - I_1$ with Tate model: $\text{ord}_u(b_i) = (1,2,3,4,5)$ and $\text{ord}_v(b_i) = (0,0,0,0,1)$

$$y^2 + b_1 uxy + b_3 u^3 = x^3 + b_2 u^2 x^2 + b_4 u^4 x + b_6 u^5 v$$

2. Non-flat Resolution: sequence of blowup

    1. $\mathbb{P}^1_i \hookrightarrow D_i \to (u=0)$: non-compact divisors

       $D_0,\ D_1, \cdots, D_8 \longleftrightarrow$ Affine roots of E_8

    2. $S_1 \longleftrightarrow$ Non-flat fiber, i.e. surface

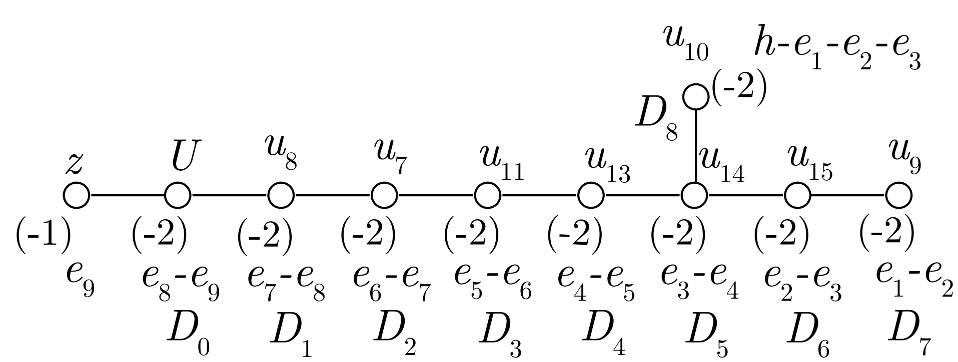
       $\Rightarrow$ Cartan of rank 1 gauge group

3. Different resolutions: S_1 contain different subset of $\mathbb{P}^1_i$

Geometrically:

- Different resolution sequences of the E_8 and codim 2 non-flat locus, yield different 5d theories.
- They are related by flops: shinking -1 curves and transforming them out of the surface S_1.
- Strategy: start with "marginal" theory, i.e. $\mathbb{P}^1_i \subset S_1$ for all $i = 0, \cdots, 8$, and descend to other models by flops.

Marginal model for rank one theories: $S_1 = gdP_9$ showing curves $\mathbb{P}^1_i = S_1 \cdot D_i$



This is the fiber for the marginal 5d theory. We can proceed by computing other blowups.... or be more efficient.

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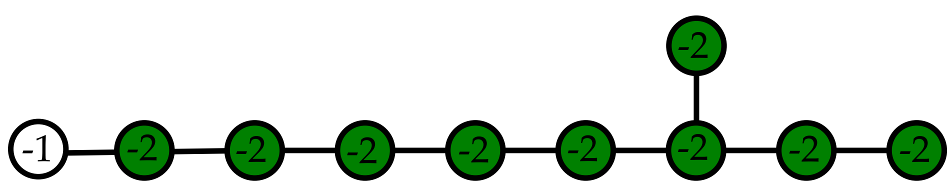

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Non-flat Resolutions as Graph-Transitions

Start with diagram of curves that are contained in non-flat fiber S_1.
For higher rank: Consider Mori cone generators of reducible surface $\cup_k S_k$.

Combined Fiber Diagram (CFD):

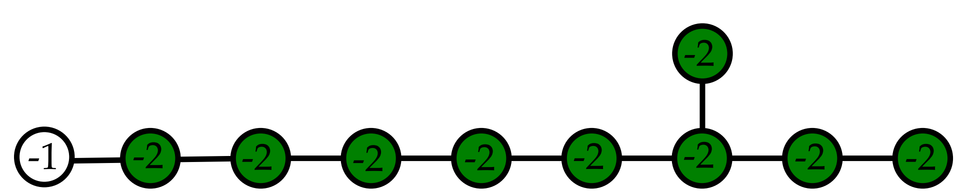
CFDs are graphs with:

Vertices: Curves C_i, labeled by $C_i^2 = n_i$

Edges: $C_i \cdot C_j = m_{i,j}$ edges connecting vertices

Marked vertices: $n_i = -2$ colored
$\Rightarrow$ subgraph = Dynkin of superconformal flavor symmetry

Marginal rank 1 CFD:



CFD Transitions

Given a CFD, the descendant CFDs are obtained by removing $n_i = (-1)$ vertex and updating

$$n'_j = n_j + m^2_{i,j}$$

$$m'_{j,k} = m_{j,k} + m_{i,j} m_{i,k}$$

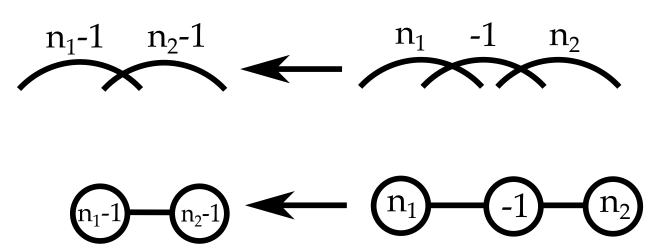
Any (-2)-vertex whose n_j changes becomes unmarked.

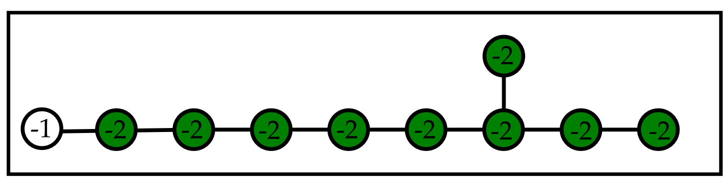
$\Rightarrow$ Network of CFDs/SCFTs

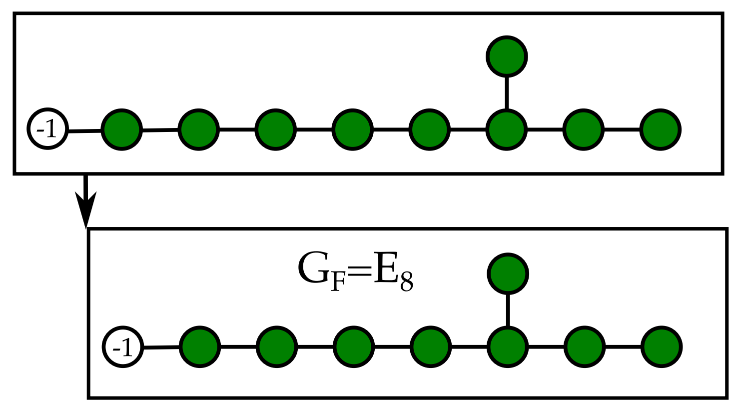
What are these?

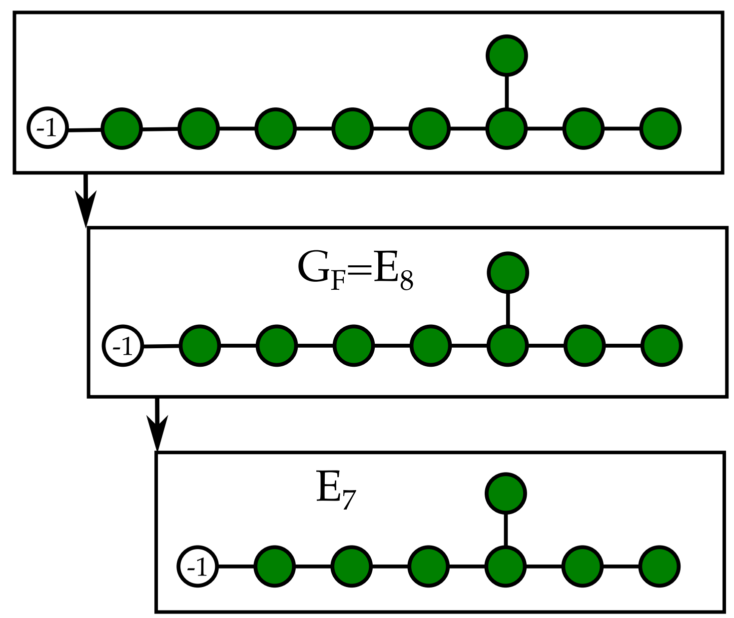
5d SCFT: gives mass to flavors and triggers RG-flow to another SCFT

Geometry: (-1) curves can be contracted and flopped out of S_i:

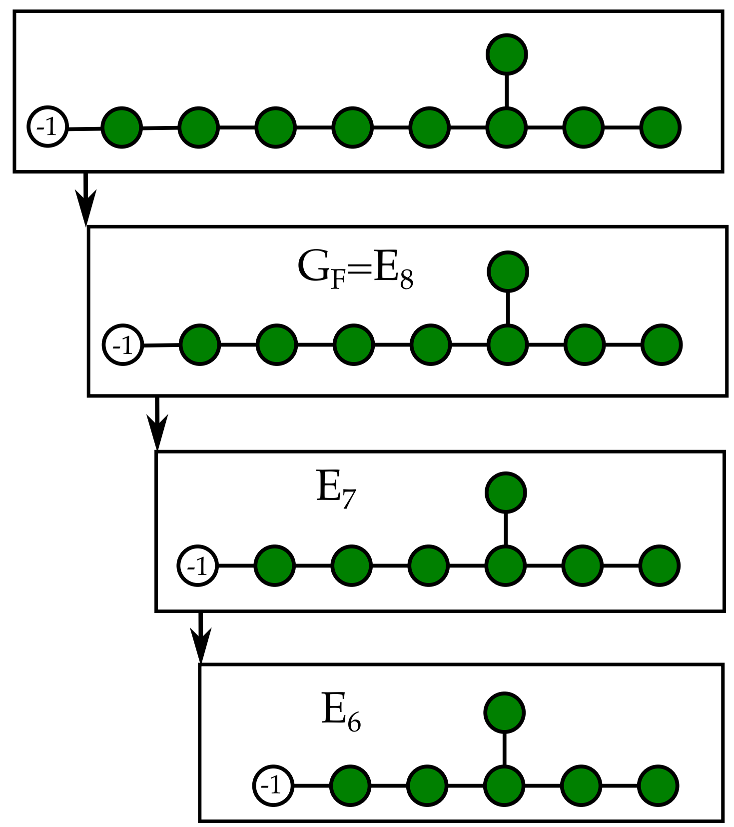


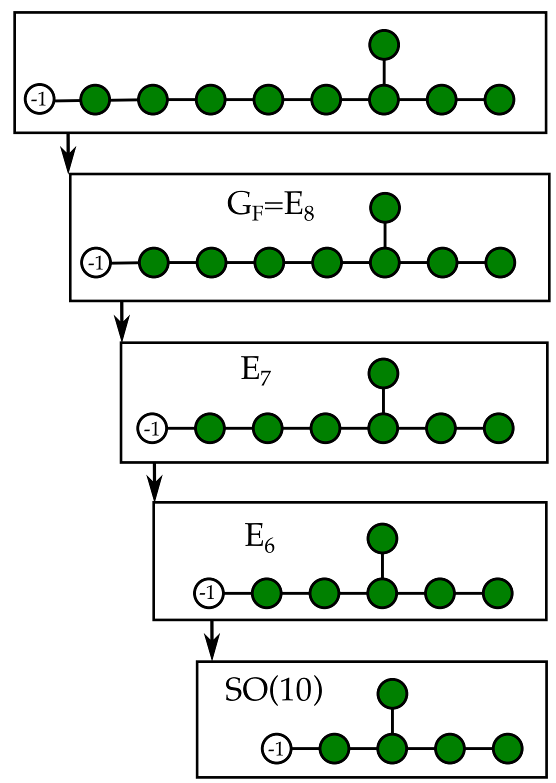

-2
-1
-2
-2
-2
-2
-2
-2
-2
-2


-1
G_F=E_8
-1

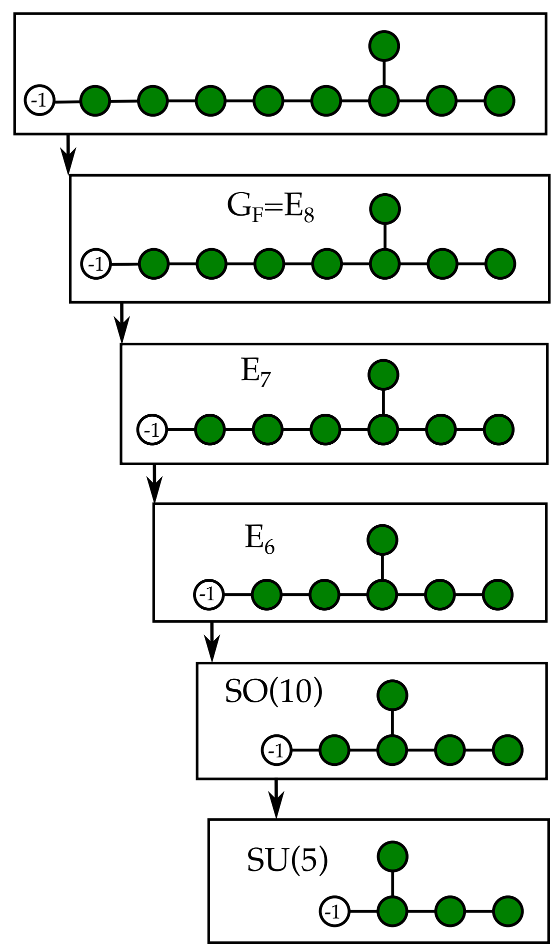


-1
G_F=E_8
-1
E_7
-1

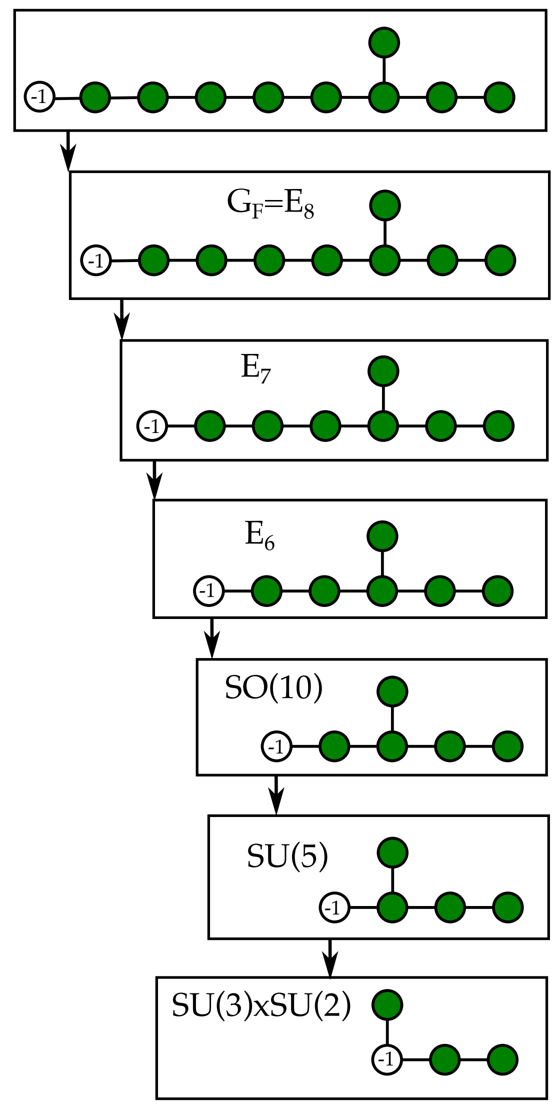

-1
G_F=E_8
-1
E_7
-1
E_6
-1



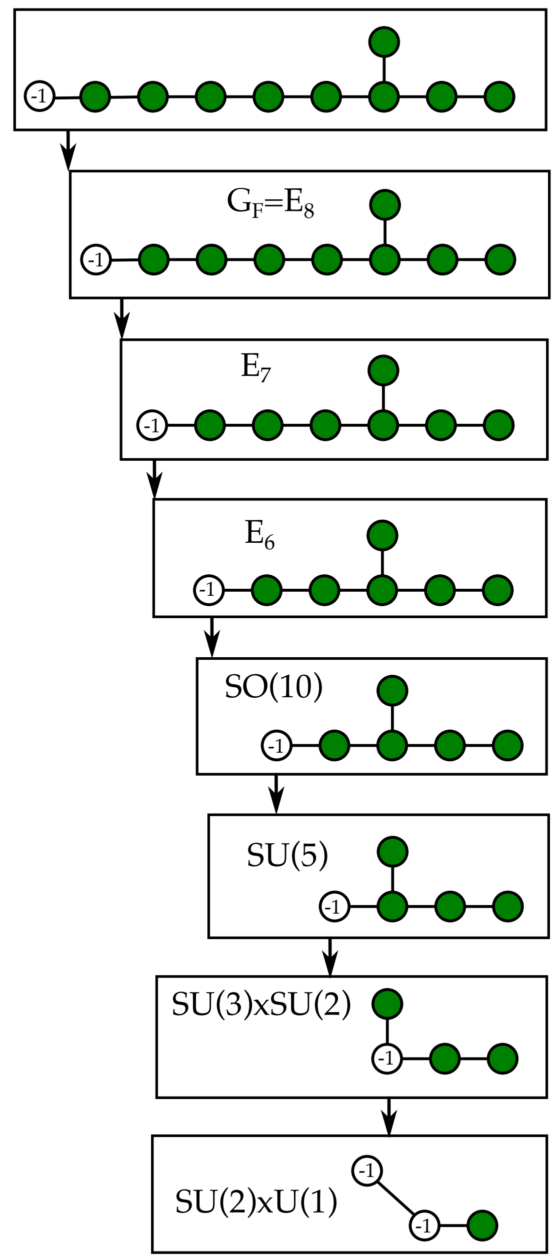
-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1



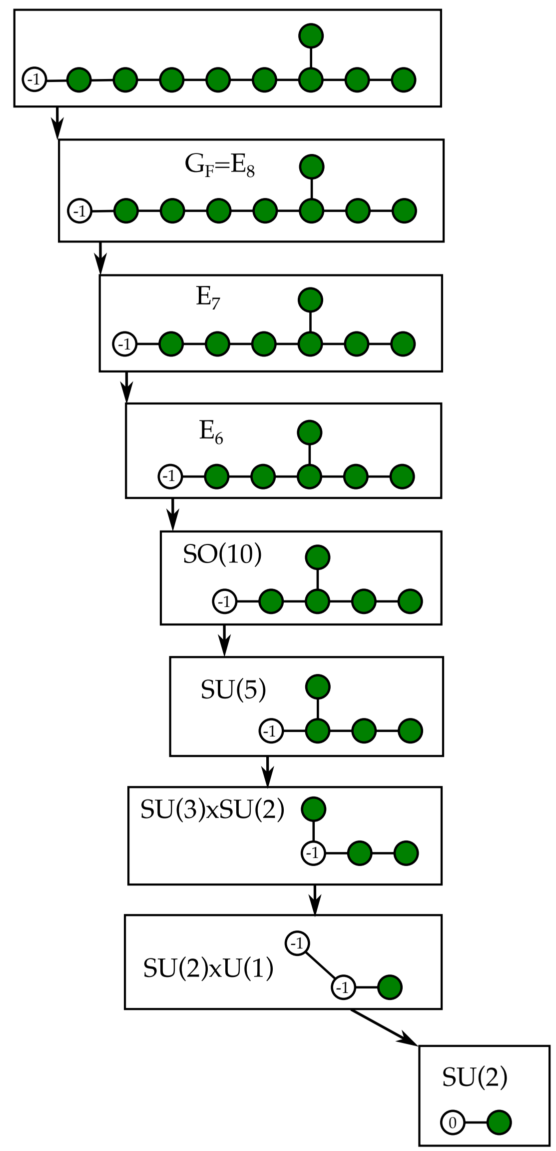
-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1
SU(5)
-1



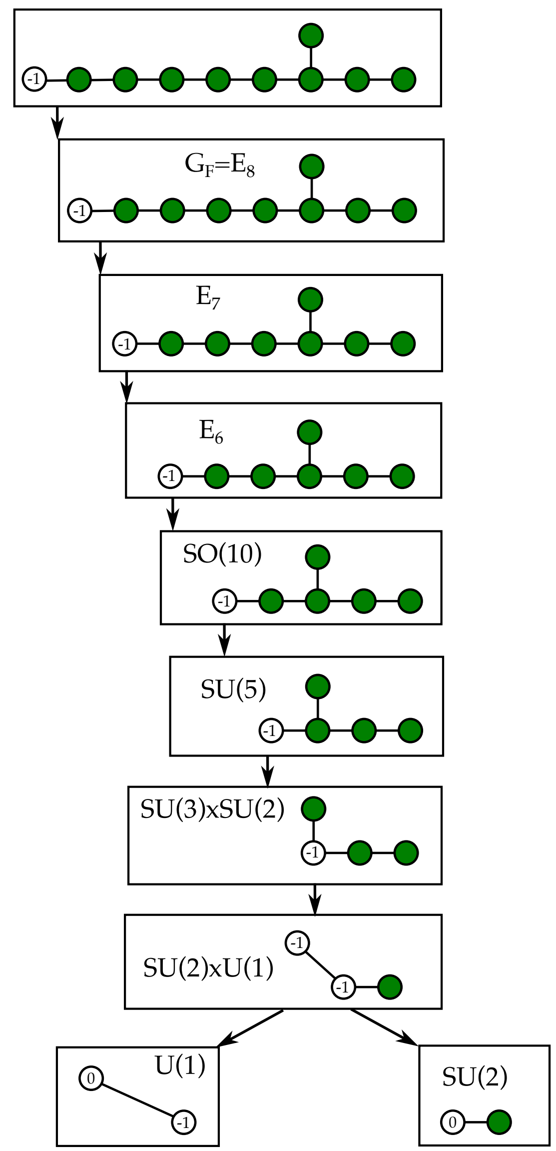
-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1
SU(5)
-1
SU(3)xSU(2)
-1



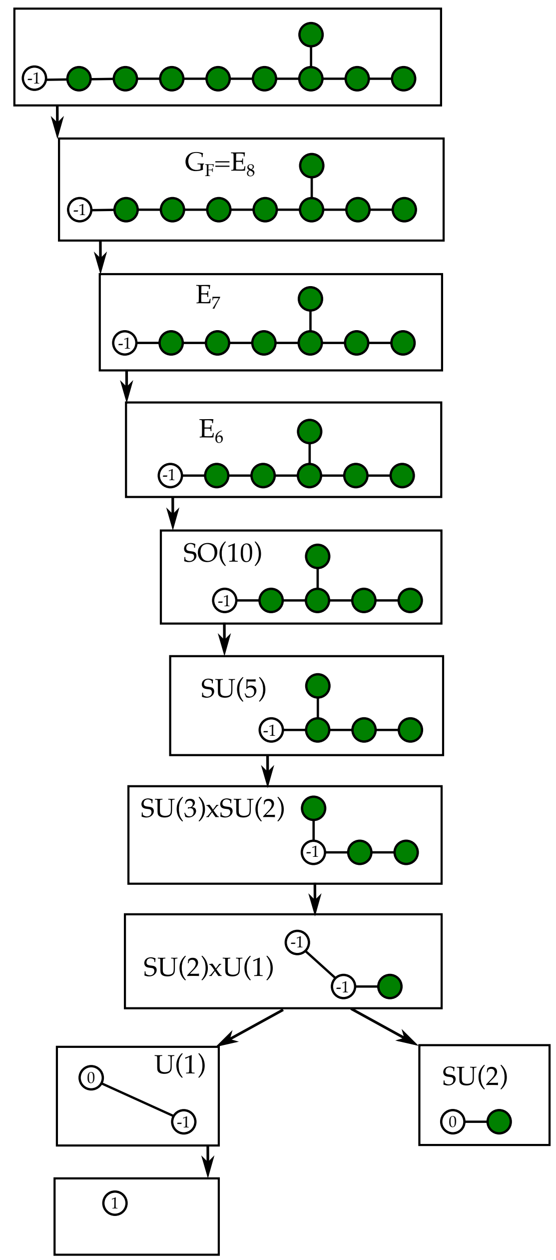
-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1
SU(5)
-1
SU(3)xSU(2)
-1
SU(2)xU(1)
-1
-1

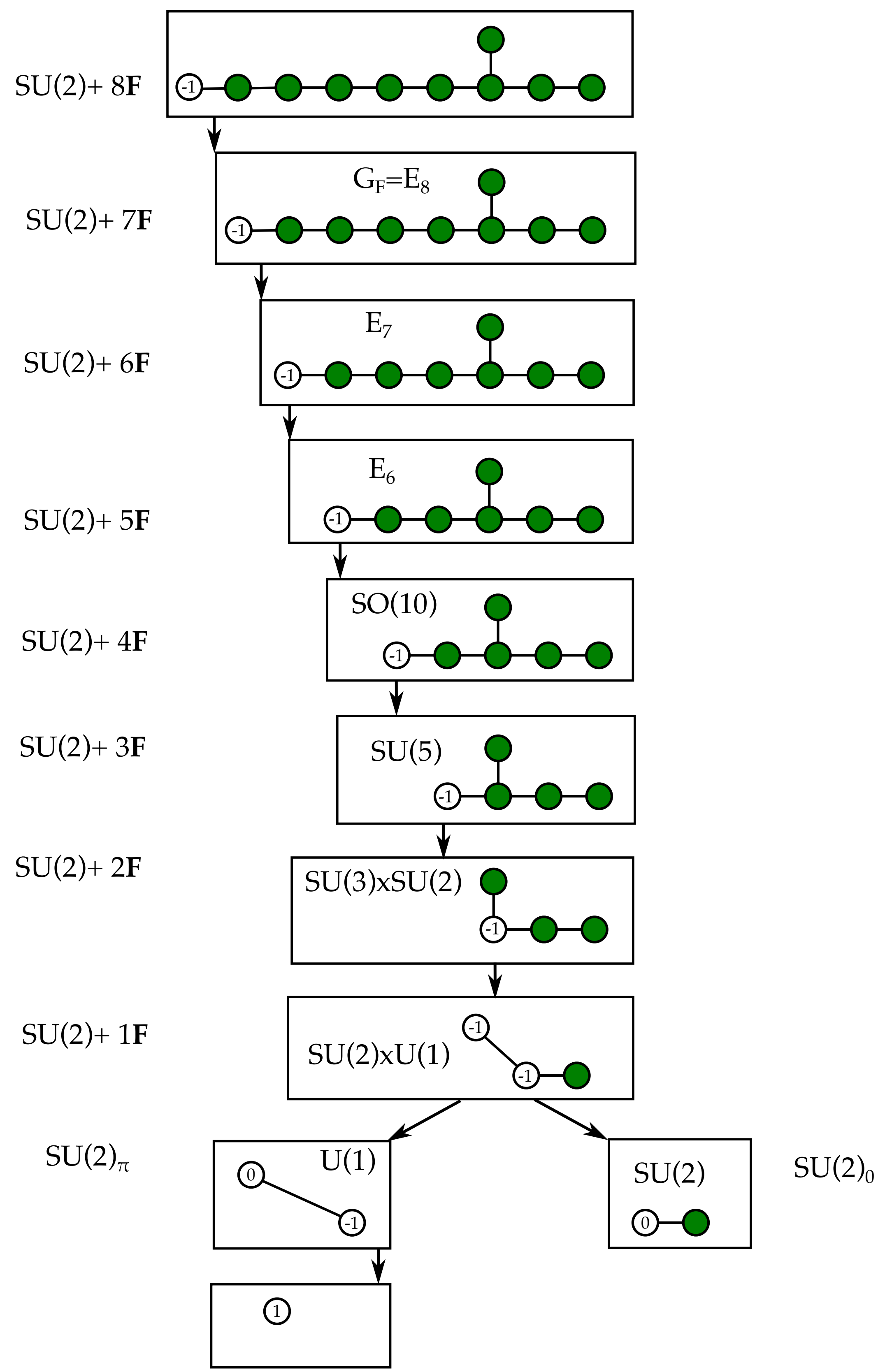


-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1
SU(5)
-1
SU(3)xSU(2)
-1
SU(2)xU(1)
-1
-1
SU(2)
0



$G_F=E_8$
E_7
E_6
SO(10)
SU(5)
SU(3)xSU(2)
SU(2)xU(1)
U(1)
SU(2)


-1
G_F=E_8
E_7
E_6
SO(10)
SU(5)
SU(3)xSU(2)
SU(2)xU(1)
U(1)
0
SU(2)
1



SU(2)+ 8F
-1
SU(2)+ 7F
$G_F=E_8$
-1
SU(2)+ 6F
E_7
-1
SU(2)+ 5F
E_6
-1
SU(2)+ 4F
SO(10)
-1
SU(2)+ 3F
SU(5)
-1
SU(2)+ 2F
SU(3)xSU(2)
-1
SU(2)+ 1F
SU(2)xU(1)
-1
-1
$SU(2)_\pi$
U(1)
0
-1
SU(2)
$SU(2)_0$
0
1

Rank 1 CFDs

This constructs precisely the known theories:

- ⋆ $SU(2)$ gauge theory with N_F flavors, which enhances to E_{N_F+1} flavor symmetry at SCFT point
  $\Rightarrow$ this reproduces known facts about rank 1 [Seiberg]
- ⋆ From non-flat fiber: $\mathbb{P}^1$s that are contained in S_1 (green) encode superconformal flavor symmetry G_F
- ⋆ Includes 5d SCFT without weakly coupled gauge theory description, geometry of $S_1 = \mathbb{P}^2$ (no ruling)

What about higher rank? Rank 2 ✓

Rank 2 theories:
geometric classification was recently obtained in [Jefferson, Katz, Kim, Vafa], and from 5-brane webs by [Hayashi, Kim, Lee, Yagi].

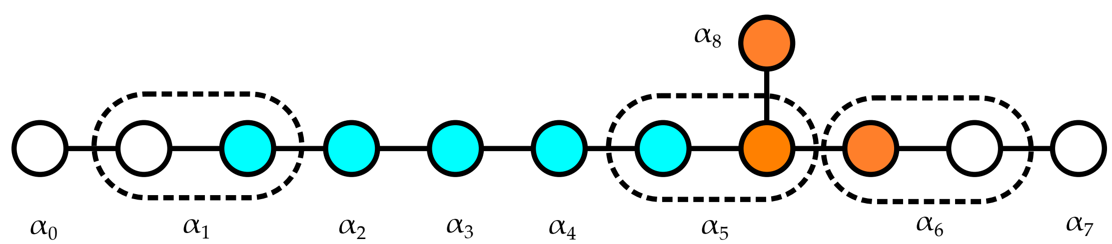
This network of SCFTs can be reconstructed from CFDs, in addition to the full flavor symmetry, and some BPS states.

Strategy: determine the marginal theories and apply CFD-transitions.

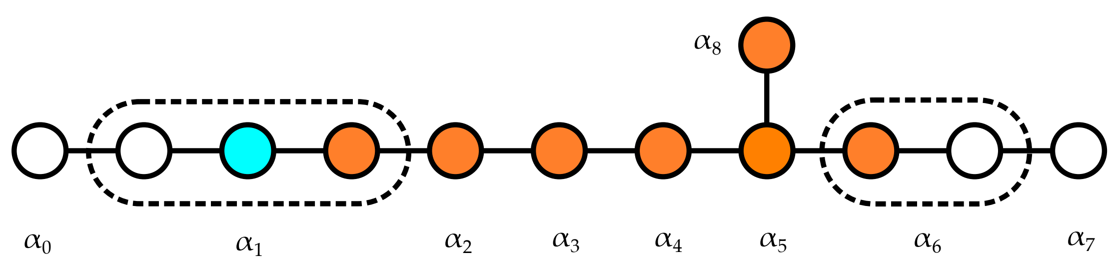
A flavor of Non-flat Resolutions: Rank 2 E-string

Codimension two collision of $E_8 - I_2$:

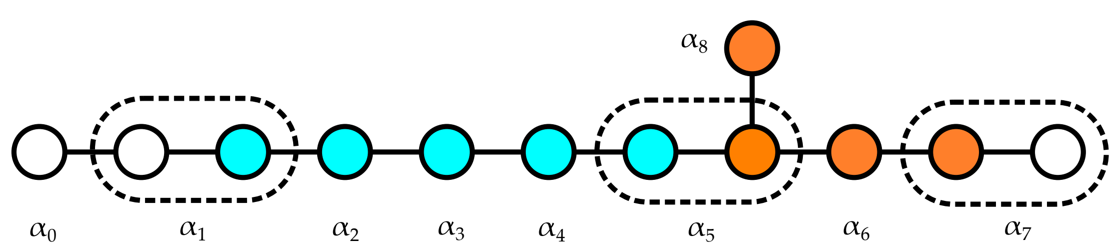
$SU(6) \times U(1)$



$SU(6) \times U(1)$



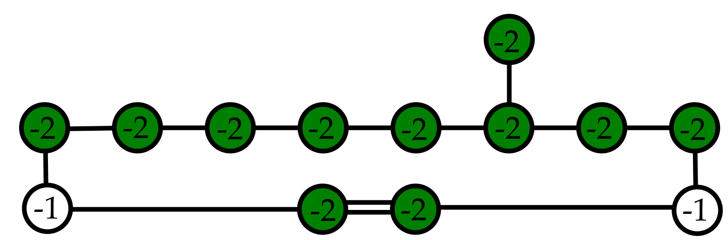
$SO(12) \times U(1)$



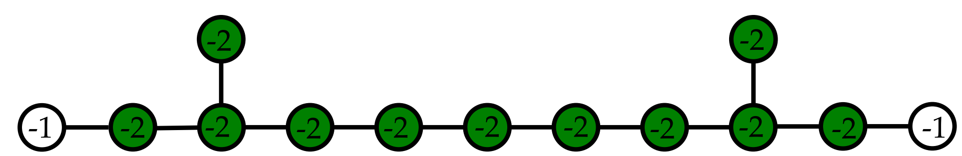
Generating all Rank 2 Theories

Rank 2 theories: marginal theories have CFDs

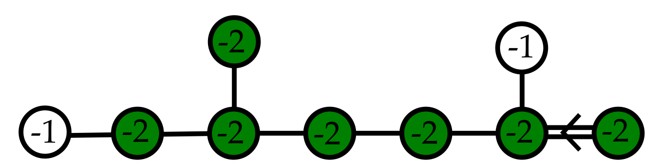
$E_8 \times SU(2)$



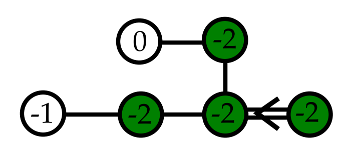
$D_5 - D_5$

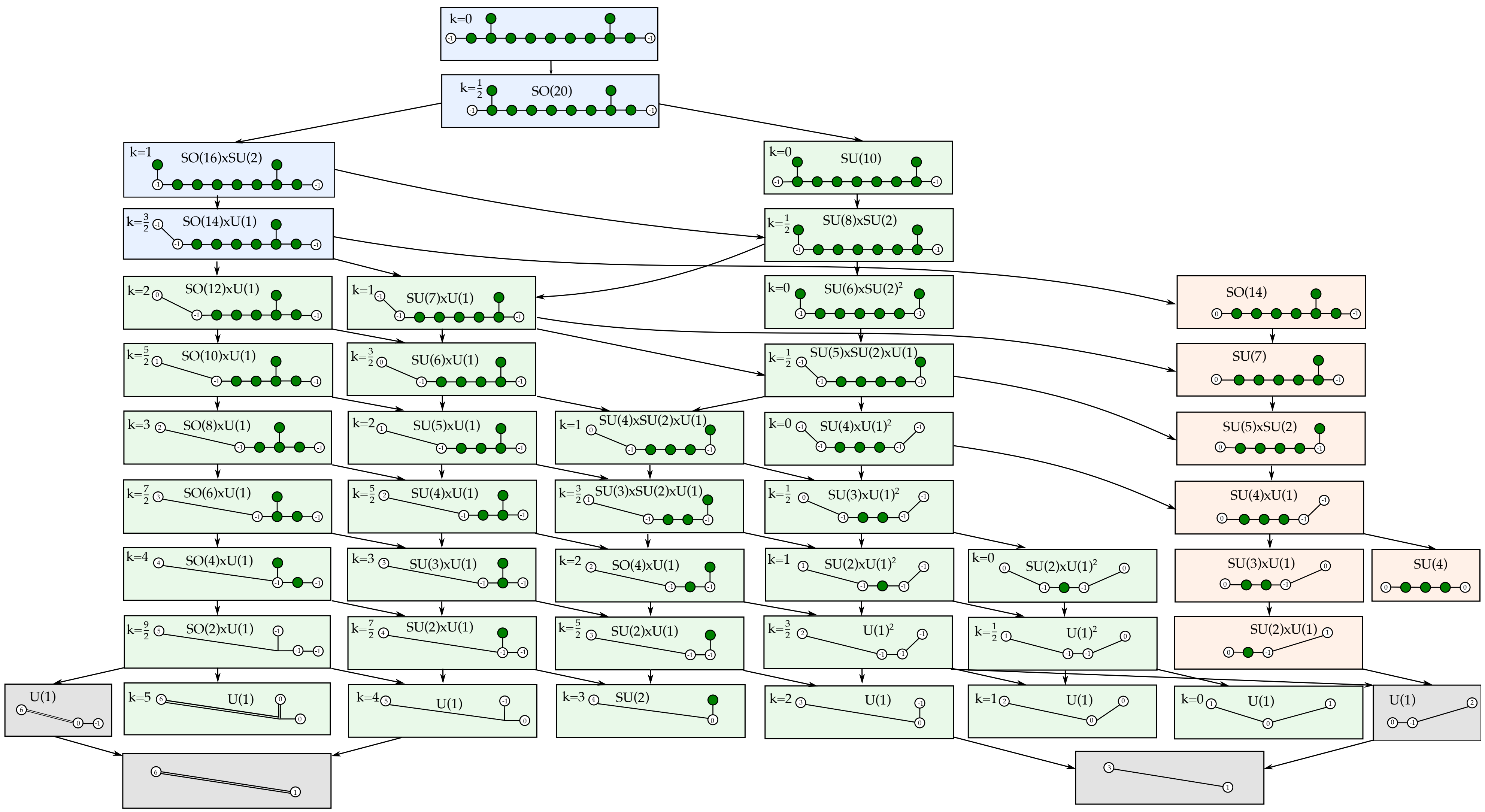


$SU(3)$ on a (-1)-curve + 12 hypers

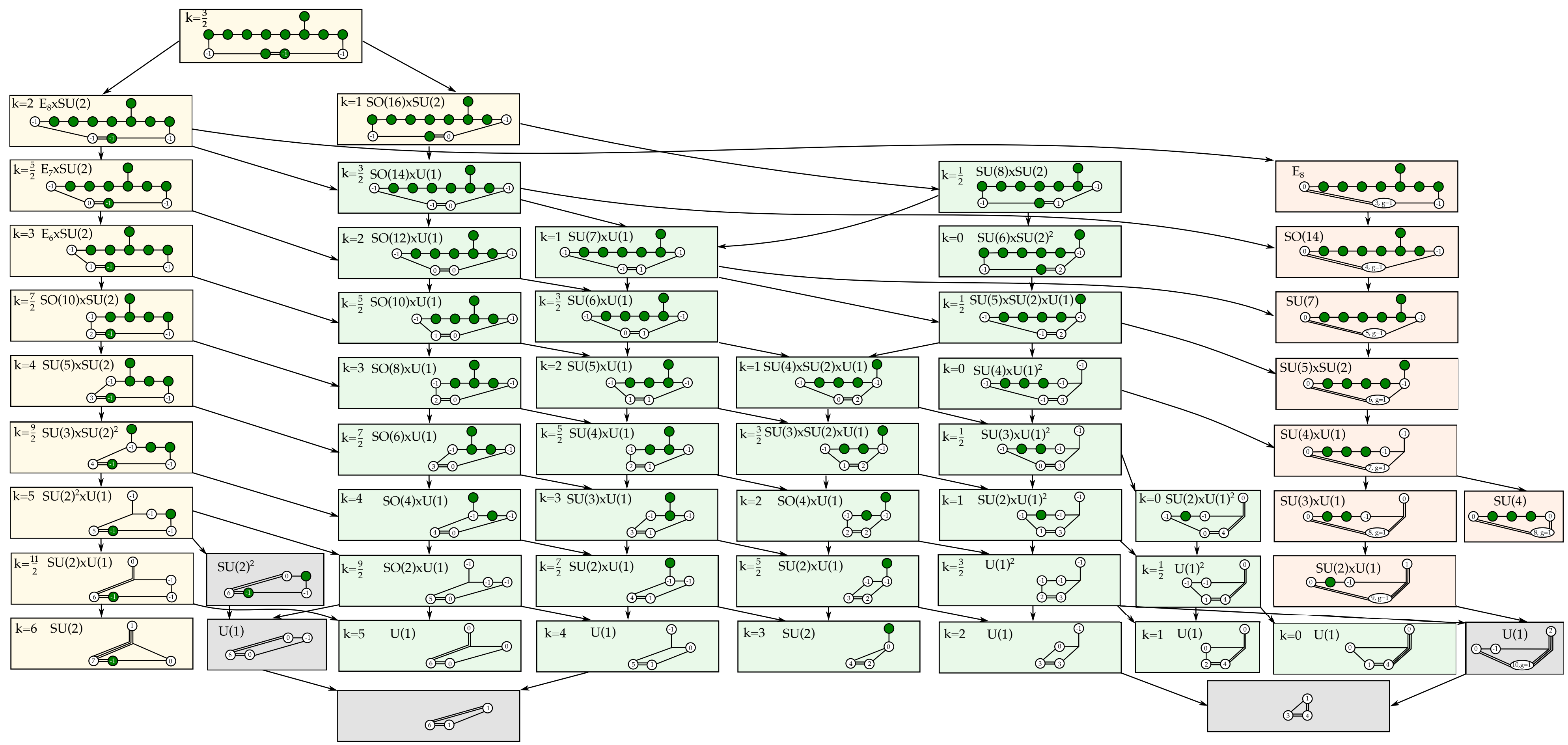


$SU(3)$ on a (-2)-curve + 6 hypers



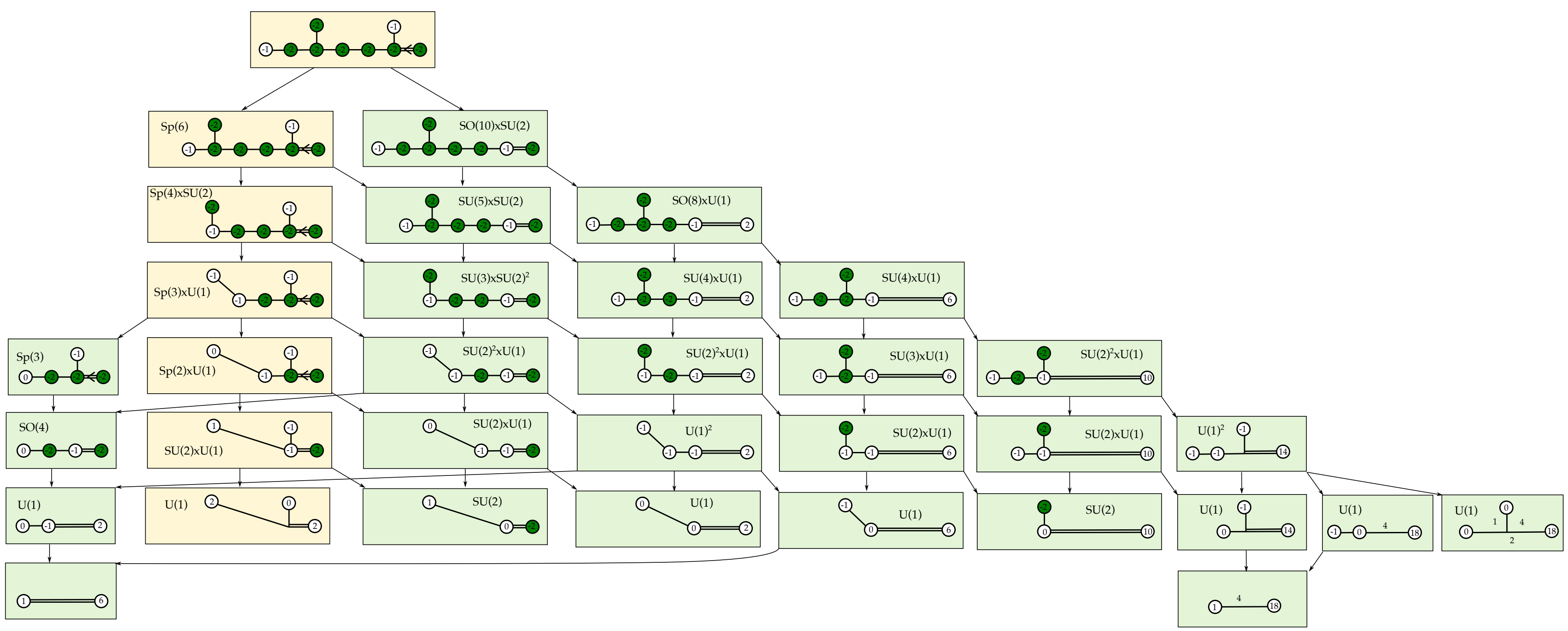


blue: only D_{10} realization; green: also rank 2 E-string realization; levels: $SU(3)$; pink: $SU(2)^2$; grey: no weakly-coupled gauge theory realization.

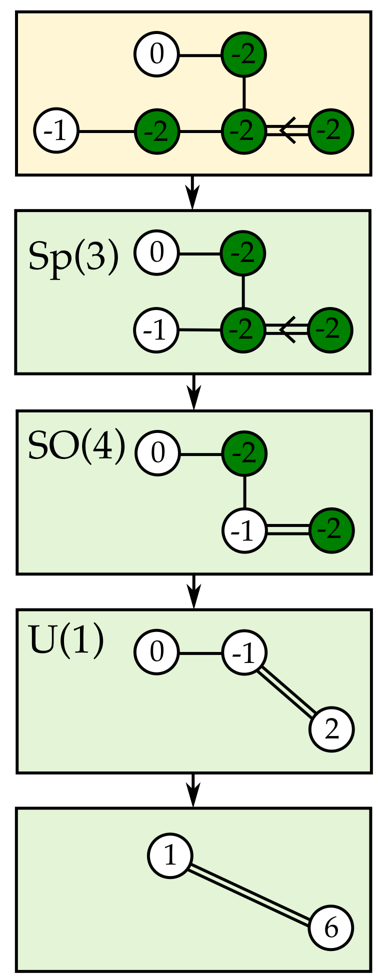


k=3/2
k=2 E8xSU(2)
k=1 SO(16)xSU(2)
k=5/2 E7xSU(2)
k=3/2 SO(14)xU(1)
k=1/2 SU(8)xSU(2)
E8
k=3 E6xSU(2)
k=2 SO(12)xU(1)
k=1 SU(7)xU(1)
k=0 SU(6)xSU(2)^2
SO(14)
k=7/2 SO(10)xSU(2)
k=5/2 SO(10)xU(1)
k=3/2 SU(6)xU(1)
k=1/2 SU(5)xSU(2)xU(1)
SU(7)
k=4 SU(5)xSU(2)
k=3 SO(8)xU(1)
k=2 SU(5)xU(1)
k=1 SU(4)xSU(2)xU(1)
k=0 SU(4)xU(1)^2
SU(5)xSU(2)
k=9/2 SU(3)xSU(2)^2
k=7/2 SO(6)xU(1)
k=5/2 SU(4)xU(1)
k=3/2 SU(3)xSU(2)xU(1)
k=1/2 SU(3)xU(1)^2
SU(4)xU(1)
k=5 SU(2)^2xU(1)
k=4 SO(4)xU(1)
k=3 SU(3)xU(1)
k=2 SO(4)xU(1)
k=1 SU(2)xU(1)^2
k=0 SU(2)xU(1)^2
SU(3)xU(1)
SU(4)
k=11/2 SU(2)xU(1)
SU(2)^2
k=9/2 SO(2)xU(1)
k=7/2 SU(2)xU(1)
k=5/2 SU(2)xU(1)
k=3/2 U(1)^2
k=1/2 U(1)^2
SU(2)xU(1)
k=6 SU(2)
U(1)
k=5 U(1)
k=4 U(1)
k=3 SU(2)
k=2 U(1)
k=1 U(1)
k=0 U(1)
U(1)

$SU(3)$ on a (-1)-curve + 12 hypers



$SU(3)$ on a (-2)-curve + 6 hypers



Cross-checks:

- Rank 1/2: complete agreement with expected network and enhanced flavor symmetry.
- Only resolution necessary to determine the CFD of the marginal theory.
- Cross-checks 1: Explicit geometric non-flat resolutions confirms the models in low rank (using methods from [Lawrie, SSN])
- Cross-checks 2: Models with weakly coupled gauge theory description [Apruzzi, Lawrie, Lin, SSN, Wang, to appear Part I] reconstruct all fibers (as well as all gauge theory phases) from systematic analysis of the Coulomb branch using methods of [Hayashi, Lawrie, Morrison SSN]: for rank 2: $SU(3)$, $Sp(2)$ and $SU(2) \times SU(2)$ gauge theory descriptions. [Apruzzi, Lawrie, Lin, SSN, Wang, to appear Part II]

BPS States

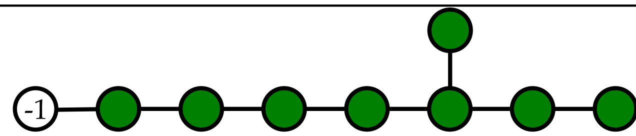
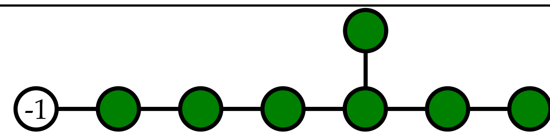
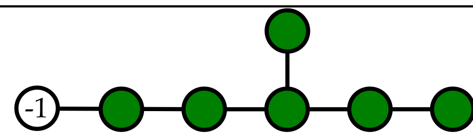
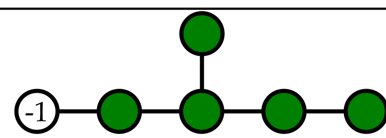
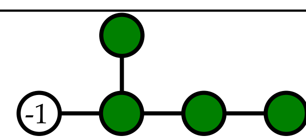
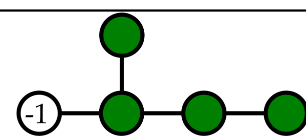
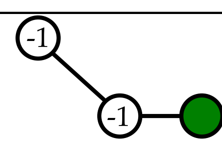
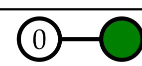
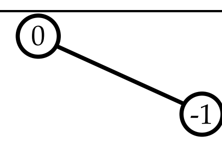
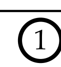
BPS states of 5d gauge theories arise as M2-branes wrapped on curves C. For rational curves, i.e. $g(C) = 0$, the BPS states transforms under the 5d massive little group $SO(4)$ as

$$R_n = \left(\frac{n}{2}, \frac{1}{2}\right) \oplus 2\left(\frac{n}{2}, 0\right) \,,$$

n=dimension of the moduli space $\mathcal{M}_C$ [Gopakumar, Vafa].

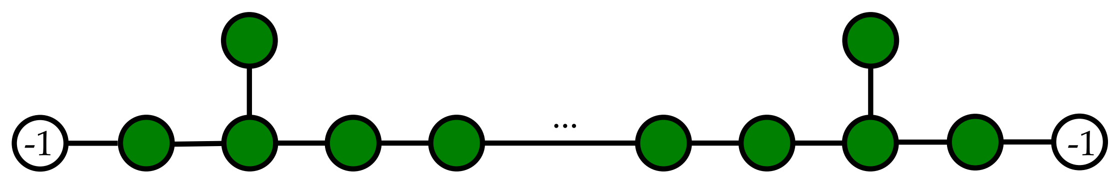
Here: C= non-negative linear combination of curves in the CFD.

For $n = 0$ 'spin 0' states we find e.g. for the E_n theories (rank 1): agreeing with [Huang, Klemm,Poretschkin]

CFD for SCFT	SCFT Flavor	Gauge Theory	BPS Spin 0
	E_8	$SU(2)+7\mathbf{F}$	**248**
	E_7	$SU(2)+6\mathbf{F}$	**56**
	E_6	$SU(2)+5\mathbf{F}$	**27**
	$SO(10)$	$SU(2)+4\mathbf{F}$	**16**
	$SU(5)$	$SU(2)+3\mathbf{F}$	**10**
	$SU(3)\times SU(2)$	$SU(2)+2\mathbf{F}$	$(\mathbf{3},\mathbf{2})$
	$SU(2)\times U(1)$	$SU(2)+1\mathbf{F}$	$\mathbf{1}_{-1},\mathbf{2}_1$
	$SU(2)$	$SU(2)_0$	
	$U(1)$	$SU(2)_\pi$	1
	-	-	

Higher Rank? Sure, all we need is the Marginal CFD

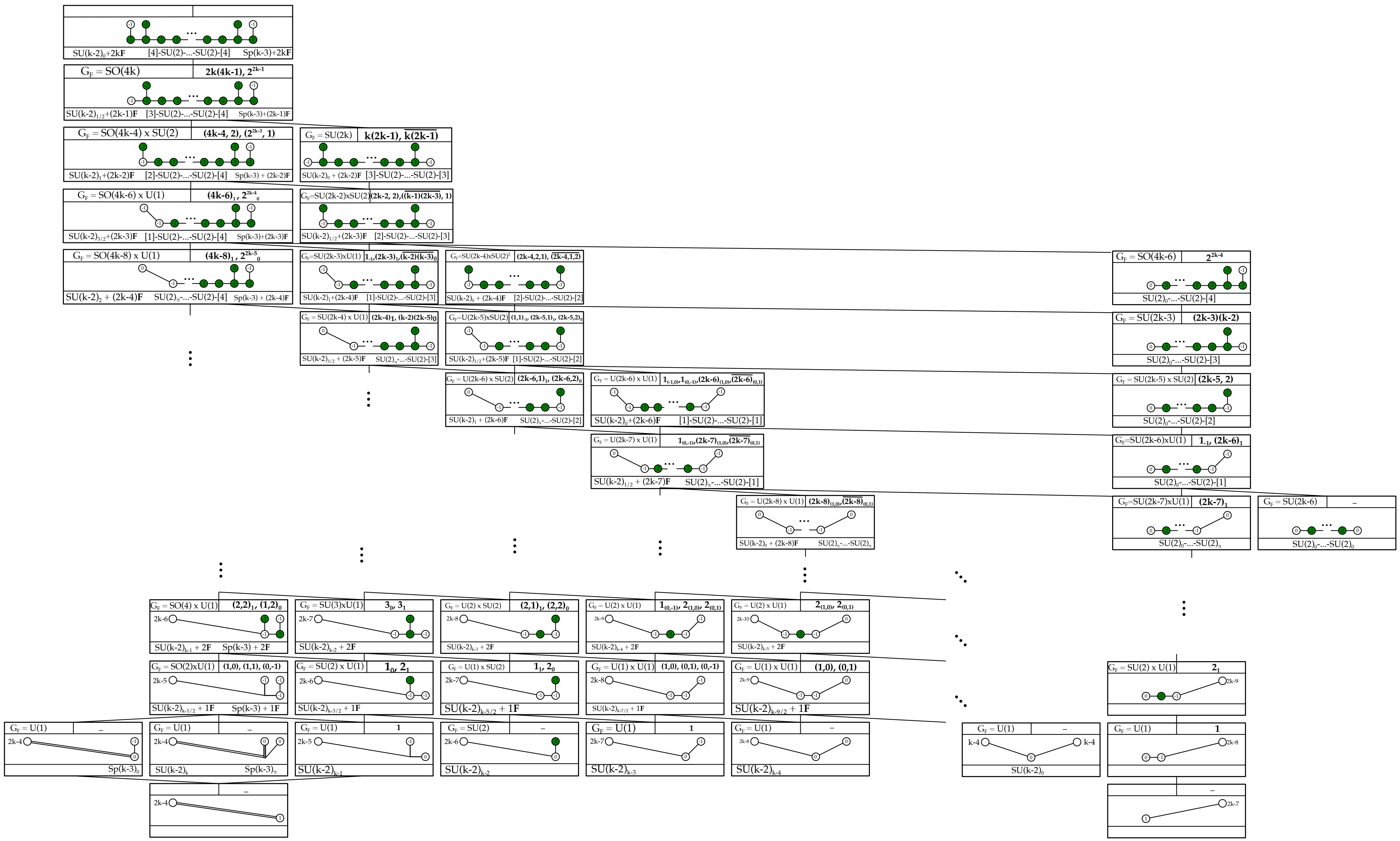
$D_k - D_k$ minimal Conformal Matter. Marginal CFD is



Weakly coupled gauge theory descriptions of marginal theory:

- $SU(k-2)_0$ with $2k\mathbf{F}$
- $4\mathbf{F} - SU(2) - ... - SU(2) - 4\mathbf{F}$, with $k-5$ $SU(2)$s nodes and theta angle 0
- $Sp(k-3)$ with $2k\mathbf{F}$.

To determine the daughter CFDs, run algorithm:



SU(k-2)0+2kF
[4]-SU(2)-...-SU(2)-[4]
Sp(k-3)+2kF
GF = SO(4k)
2k(4k-1), 2^{2k-1}
SU(k-2)1/2+(2k-1)F
[3]-SU(2)-...-SU(2)-[4]
Sp(k-3)+(2k-1)F
GF = SO(4k-4) x SU(2)
(4k-4, 2), (2^{2k-3}, 1)
SU(k-2)1+(2k-2)F
[2]-SU(2)-...-SU(2)-[4]
Sp(k-3) + (2k-2)F
GF = SU(2k)
k(2k-1), k(2k-1)
SU(k-2)0 + (2k-2)F
[3]-SU(2)-...-SU(2)-[3]
GF = SO(4k-6) x U(1)
(4k-6)1, 2^{2k-4}0
SU(k-2)3/2+(2k-3)F
[1]-SU(2)-...-SU(2)-[4]
Sp(k-3)+(2k-3)F
GF=SU(2k-2)xSU(2)
(2k-2, 2), ((k-1)(2k-3), 1)
SU(k-2)1/2+(2k-3)F
[2]-SU(2)-...-SU(2)-[3]
GF = SO(4k-8) x U(1)
(4k-8)1, 2^{2k-5}0
SU(k-2)2 + (2k-4)F
SU(2)π-...-SU(2)-[4]
Sp(k-3) + (2k-4)F
GF=SU(2k-3)xU(1)
SU(k-2)1+(2k-4)F
[1]-SU(2)-...-SU(2)-[3]
GF=SU(2k-4)xSU(2)^2
(2k-4,2,1), (2k-4,1,2)
SU(k-2)0 + (2k-4)F
[2]-SU(2)-...-SU(2)-[2]
GF = SO(4k-6)
2^{2k-4}
SU(2)0-...-SU(2)-[4]
GF = SU(2k-4) x U(1)
SU(k-2)3/2 + (2k-5)F
SU(2)π-...-SU(2)-[3]
GF=U(2k-5)xSU(2)
SU(k-2)1/2+(2k-5)F
[1]-SU(2)-...-SU(2)-[2]
GF = SU(2k-3)
(2k-3)(k-2)
SU(2)0-...-SU(2)-[3]
GF = U(2k-6) x SU(2)
(2k-6,1)1, (2k-6,2)0
SU(k-2)1 + (2k-6)F
SU(2)π-...-SU(2)-[2]
GF = U(2k-6) x U(1)
SU(k-2)0+(2k-6)F
[1]-SU(2)-...-SU(2)-[1]
GF = SU(2k-5) x SU(2)
(2k-5, 2)
SU(2)0-...-SU(2)-[2]
GF = U(2k-7) x U(1)
SU(k-2)1/2 + (2k-7)F
SU(2)π-...-SU(2)-[1]
GF=SU(2k-6)xU(1)
SU(2)0-...-SU(2)-[1]
GF = U(2k-8) x U(1)
SU(k-2)0 + (2k-8)F
SU(2)π-...-SU(2)π
GF=SU(2k-7)xU(1)
(2k-7)1
SU(2)0-...-SU(2)π
GF = SU(2k-6)
SU(2)0-...-SU(2)0
GF = SO(4) x U(1)
(2,2)1, (1,2)0
SU(k-2)k-1 + 2F
Sp(k-3) + 2F
GF = SU(3)xU(1)
30, 31
SU(k-2)k-2 + 2F
GF = U(2) x SU(2)
(2,1)1, (2,2)0
SU(k-2)k-3 + 2F
GF = U(2) x U(1)
SU(k-2)k-4 + 2F
GF = U(2) x U(1)
SU(k-2)k-5 + 2F
GF = SO(2)xU(1)
(1,0), (1,1), (0,-1)
SU(k-2)k-1/2 + 1F
Sp(k-3) + 1F
GF = SU(2) x U(1)
10, 21
SU(k-2)k-3/2 + 1F
GF = U(1) x SU(2)
11, 20
SU(k-2)k-5/2 + 1F
GF = U(1) x U(1)
(1,0), (0,1), (0,-1)
SU(k-2)k-7/2 + 1F
GF = U(1) x U(1)
(1,0), (0,1)
SU(k-2)k-9/2 + 1F
GF = SU(2) x U(1)
21
GF = U(1)
Sp(k-3)0
GF = U(1)
SU(k-2)k
Sp(k-3)π
GF = U(1)
1
SU(k-2)k-1
GF = SU(2)
SU(k-2)k-2
GF = U(1)
1
SU(k-2)k-3
GF = U(1)
SU(k-2)k-4
GF = U(1)
SU(k-2)0
GF = U(1)
1

$D_k - D_k$ cont'd.

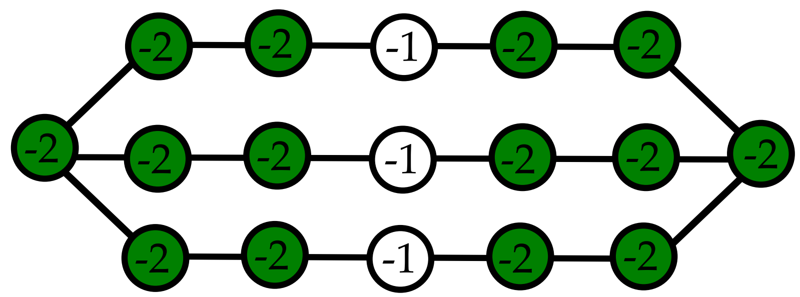
\# $(k-2)^2 - 3$ descendant SCFTs,
\# $2k-6$ without known gauge theory description.
\# Flavor enhancement, e.g. for models with $SU(k-2)_\kappa + m\mathbf{F}$ gauge descr.

$$
\begin{array}{rc}
\kappa & \text{SCFT Flavor Symmetry } G_F \\
k - \frac{m}{2} : & \begin{cases} SO(4k) & m = 2k-1 \\ SO(4k-4)\times SU(2) & m = 2k-2 \\ SO(2m)\times U(1) & m = 0, \dots, 2k-3 \end{cases} \\
k - 1 - \frac{m}{2} : & \begin{cases} SU(2k) & m = 2k-2 \\ SU(2k-2)\times SU(2) & m = 2k-3 \\ SU(m+1)\times U(1) & m = 0, \dots, 2k-4 \end{cases} \\
k - 2 - \frac{m}{2} : & \begin{cases} SU(2k-4)\times SU(2)^2 & m = 2k-4 \\ U(m)\times SU(2) & m = 0, \dots, 2k-5 \end{cases}
\end{array}
$$

In agreement with recent results in [Cabrera, Hanany, Zajac, 10/2018].

Higher Rank, cont'd

(E_6, E_6) minimal Conformal Matter (CM): marginal theory has CFD



We find 207 descendant CFDs/SCFTs, including flavor symmetry and tree structure. Only known [del Zotto, Heckman, Tomasiello, Vafa] weakly coupled quiver description

$$\begin{array}{c} [2] \\ | \\ SU(2) \\ | \\ [2] - SU(2) - \ SU(3)_0 \ - SU(2) - [2] \,. \end{array}$$

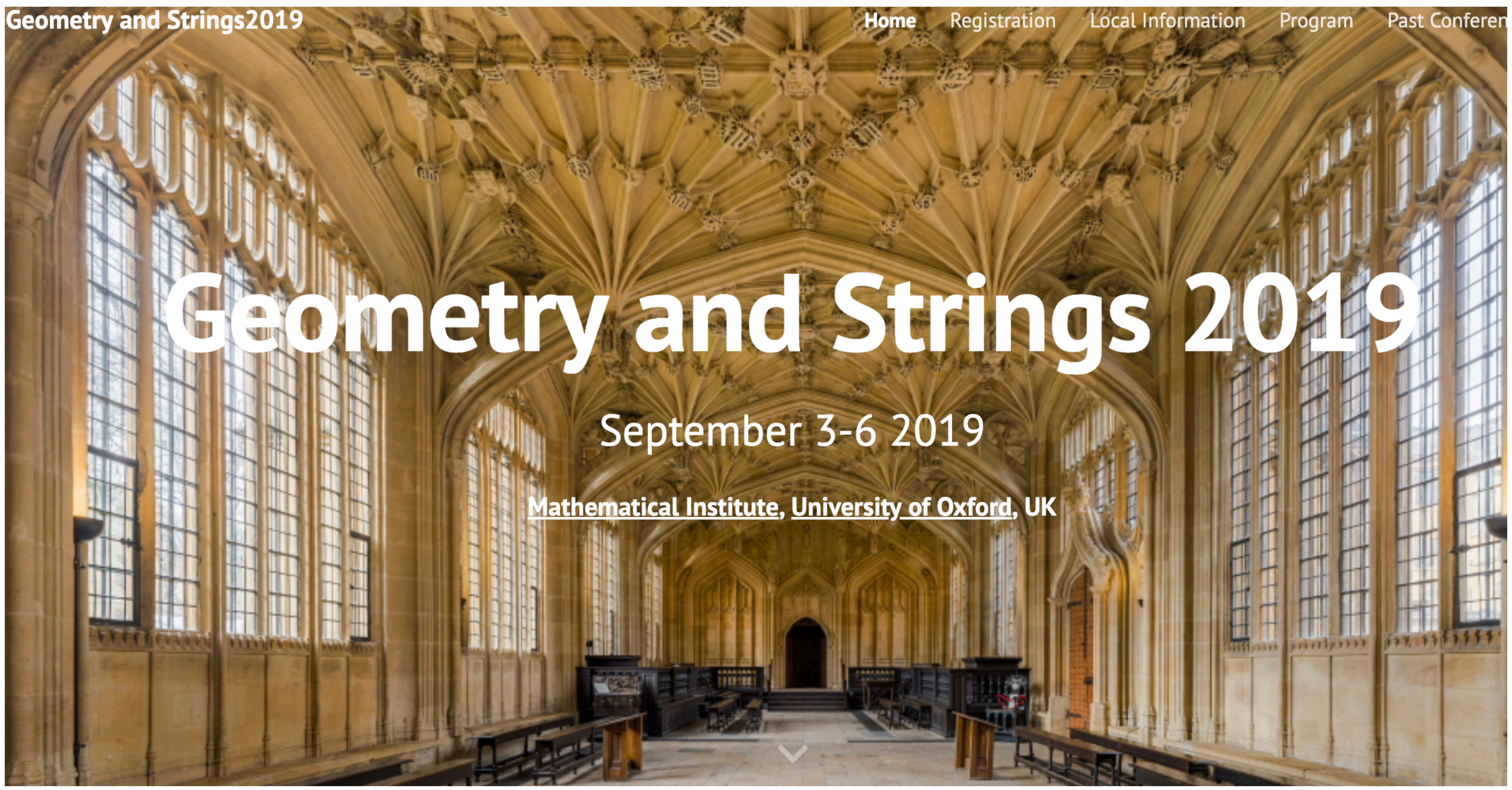
We find additional 195 theories that have no known gauge theory description. Similar structure for other CM matter theories.

.... because they add flavor to 5d SCFTs!

Summary and Outlook

- Given a 6d SCFT, we provide a systematic exploration of all descendant 5d SCFTs.
- Each 5d SCFT is characterized in terms of a CFD graph, which manifestly encodes the superconformal flavor symmetry, and spin 0 BPS states.
- Provides classification of 5d SCFTs under assumption that they all descend from 6d ones.

"Geometry and Strings 2019"
(aka Geometry and Physics of F-theory)



https://sites.google.com/view/geometryandstrings2019/home